# Compressed Sensing: What is It?

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www.jtsense.com

Electrical Engineering and Computer Sciences
University of California, Berkeley

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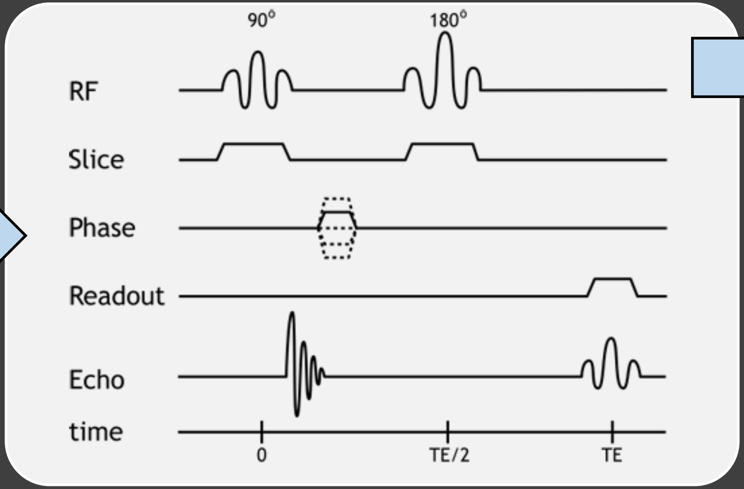
# MRI Background

Patient in MRI scanner

Pulse sequence controls MRI signal




Measurements are collected

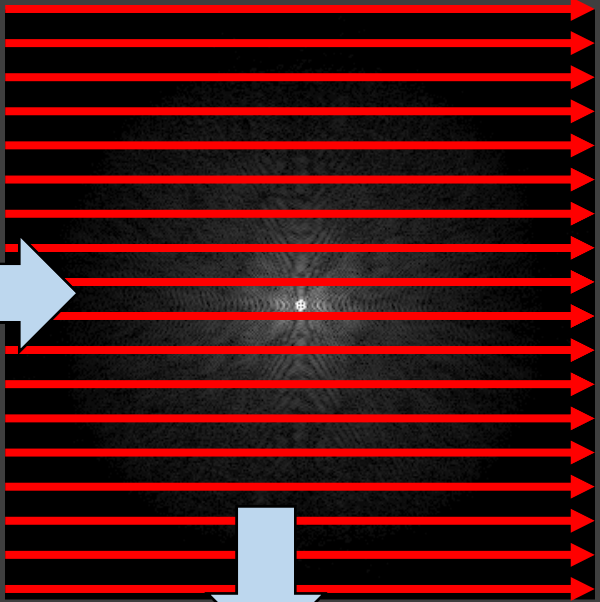
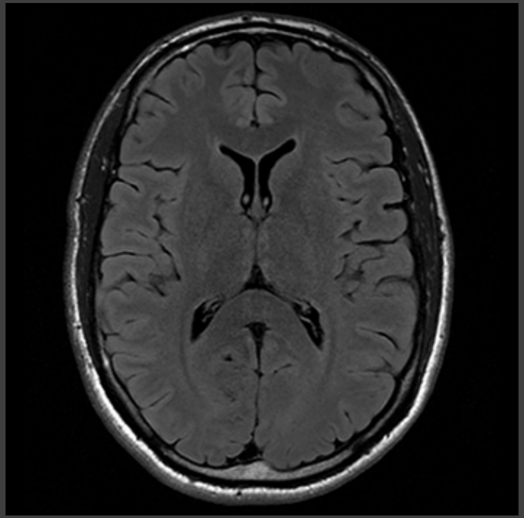


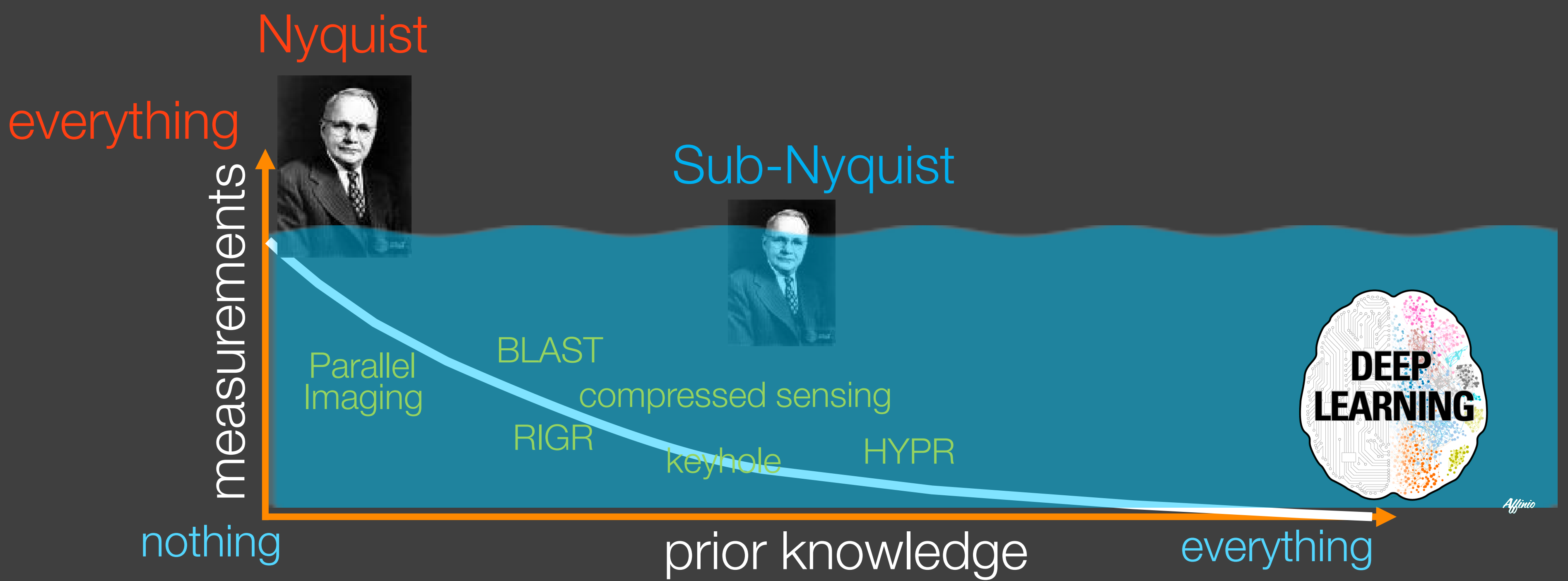
Image is reconstructed



https://www.mountnittany.org/assets/images/krames/Image_201608102119_2786.jpg
http://xrayphysics.com/se_diag_wgrad.png

# Data Redundancy

Redundancy reduces sampling requirements
(The more you know, the less you need)

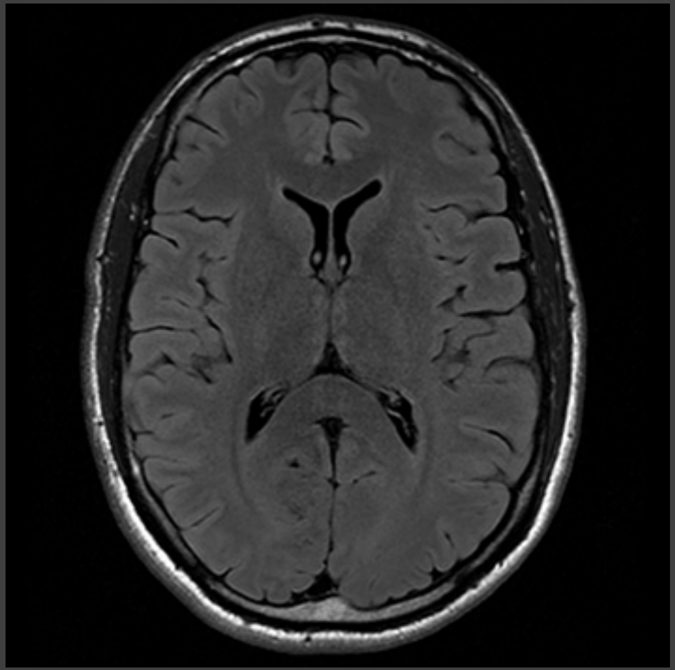



From M. Lustig

# Collecting (less) data

- Scan time is proportional to number of measurements
  - Collect less data → scan faster!
  - Under-sampling causes artifacts

image space



k-space

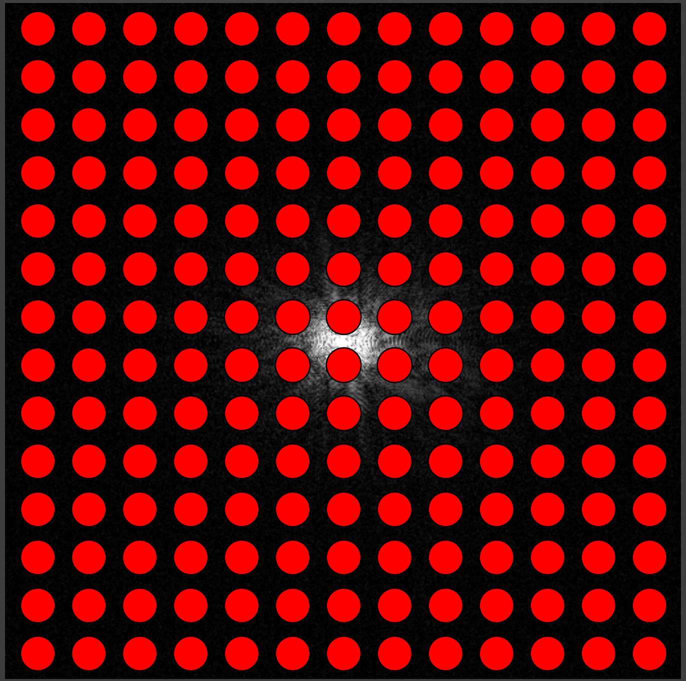


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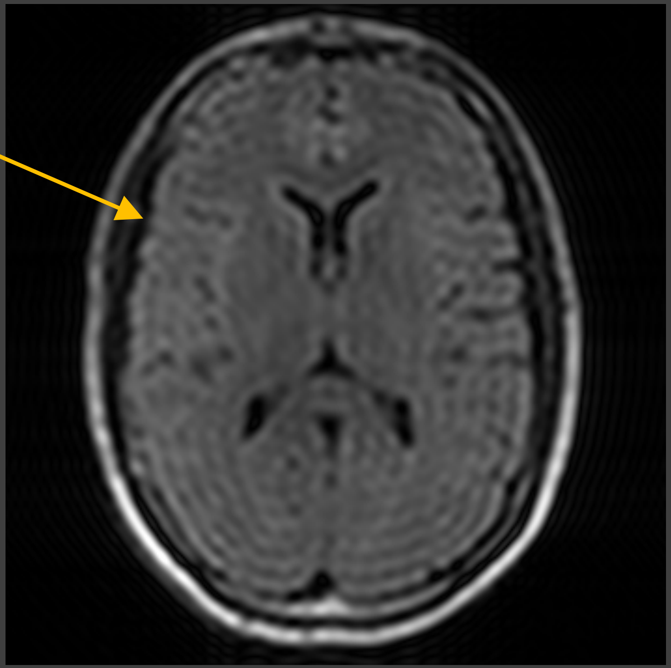
image space

Low resolution (ringing)



k-space

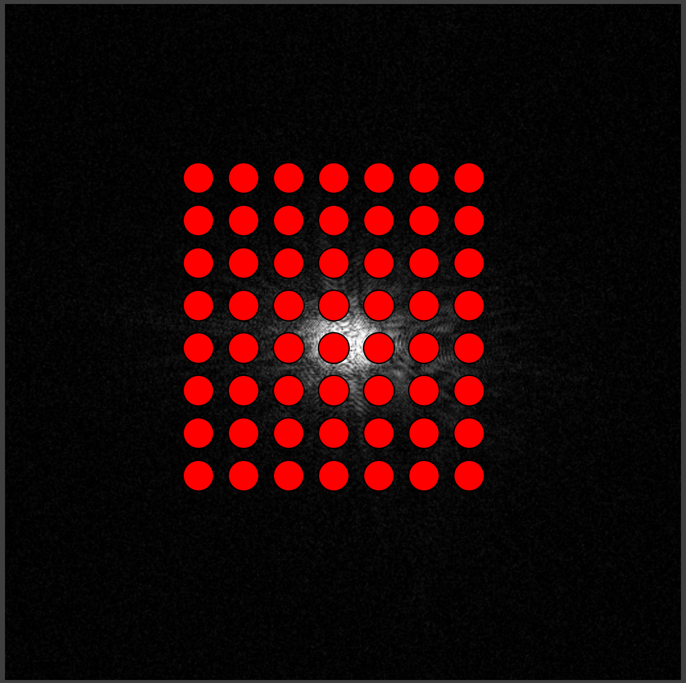


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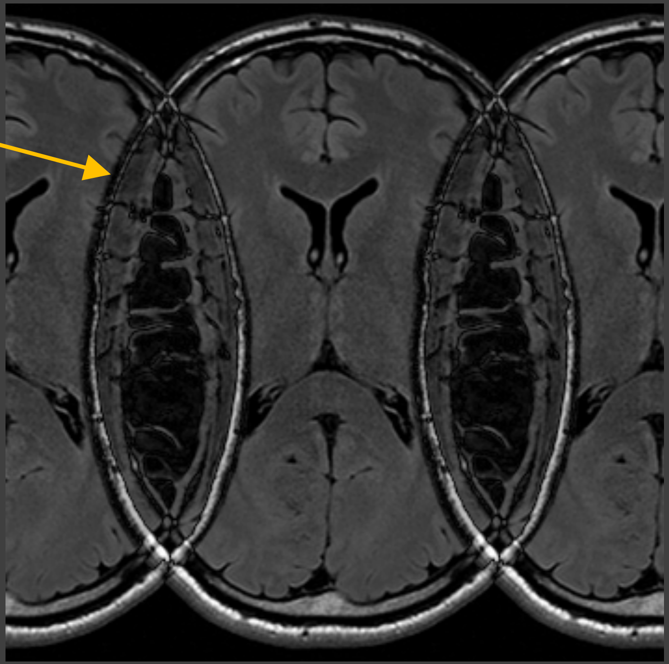
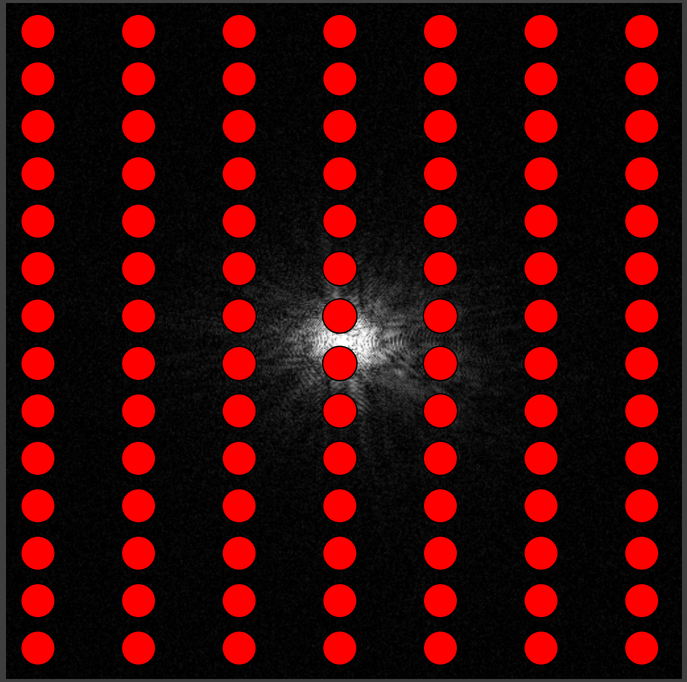
image space

Coherent aliasing



k-space

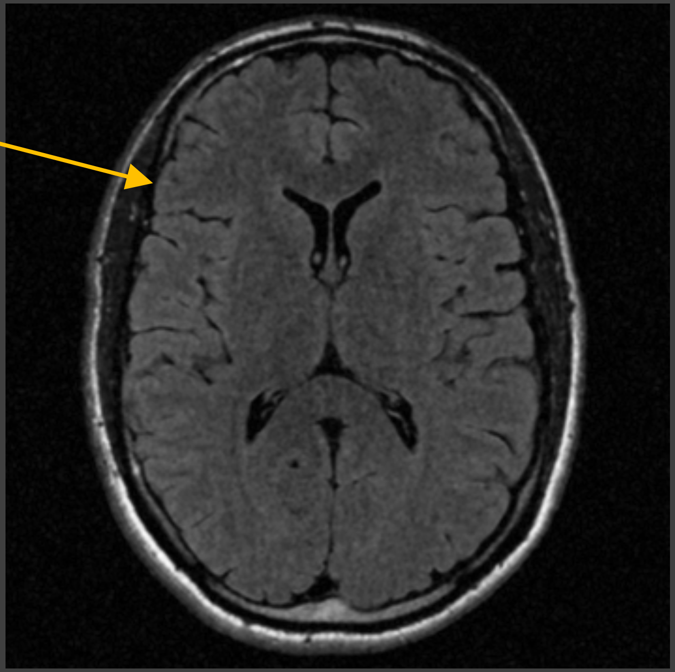


# Collecting (less) data

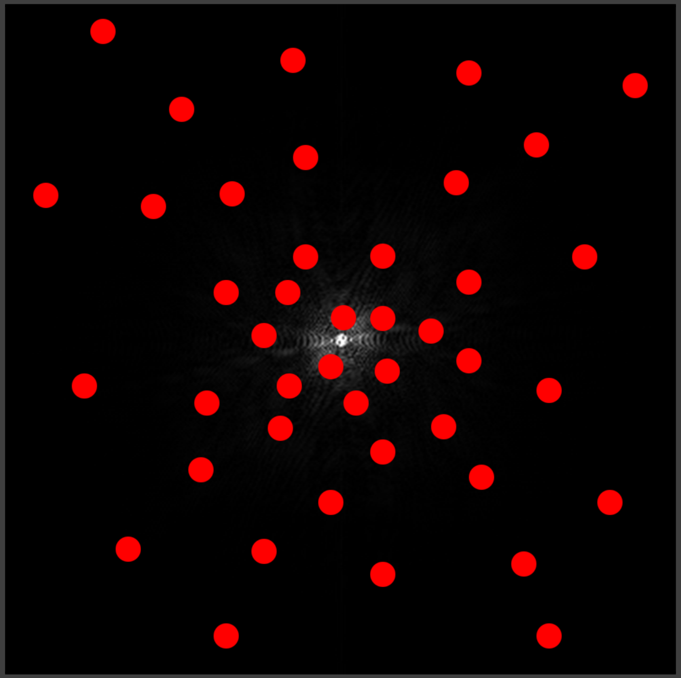
- Scan time is proportional to number of measurements
  - Collect less data → scan faster!
  - Under-sampling causes artifacts

image space

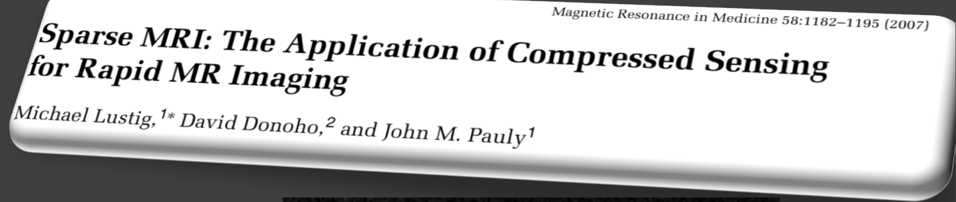
Incoherent aliasing



k-space

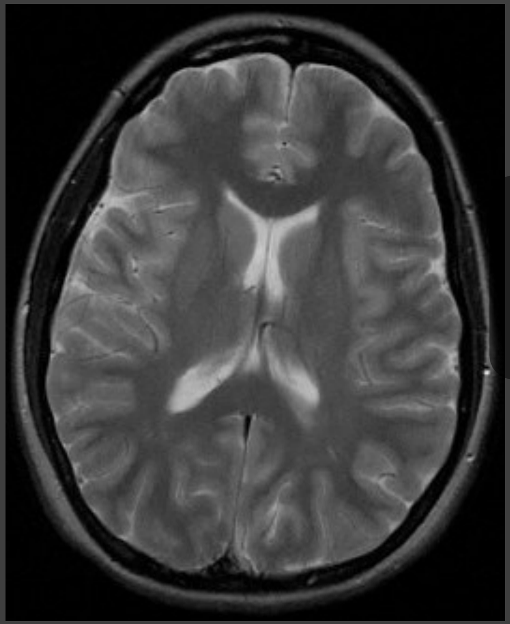


# Compressed Sensing MRI

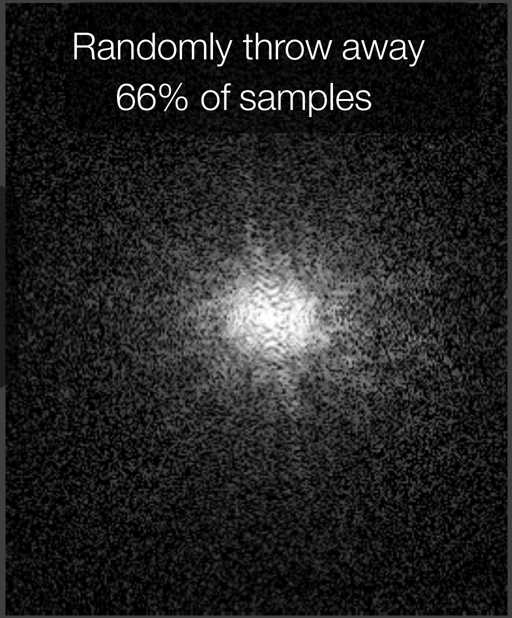

Magnetic Resonance in Medicine 58:1182–1195 (2007)

Sparse MRI: The Application of Compressed Sensing for Rapid MR Imaging

Michael Lustig,[1*] David Donoho,[2] and John M. Pauly[1]

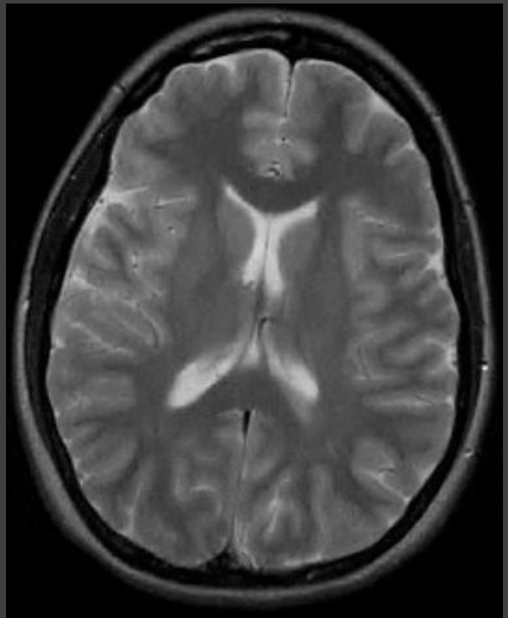


Fourier
transform




standard
recon

compressed
sensing



From M. Lustig

Magnetic Resonance in Medicine 58:1182–1195 (2007)

**Sparse MRI: The Application of Compressed Sensing for Rapid MR Imaging**

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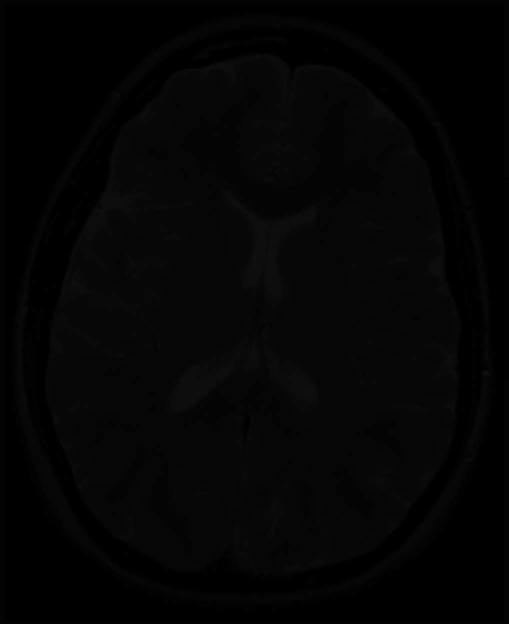
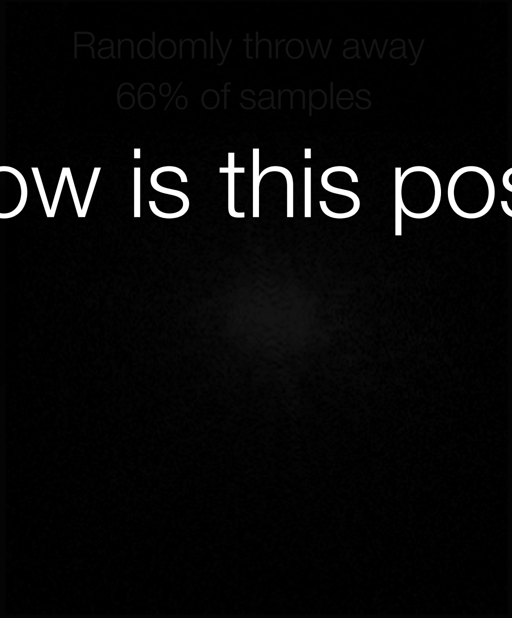
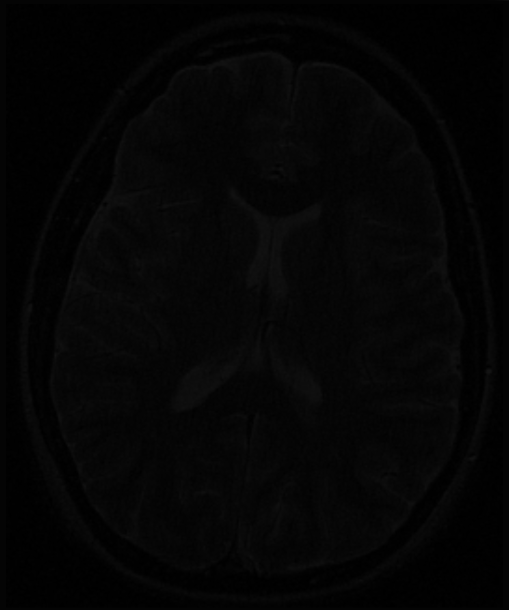
# Compressed Sensing MRI



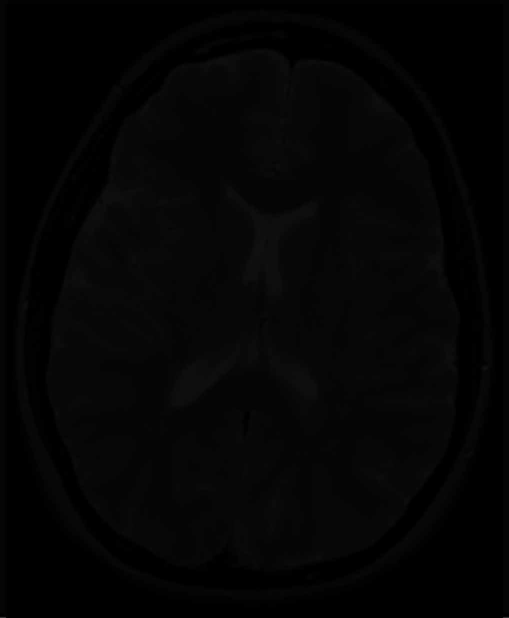
Fourier
transform




standard
recon



compressed
sensing



## How is this possible?

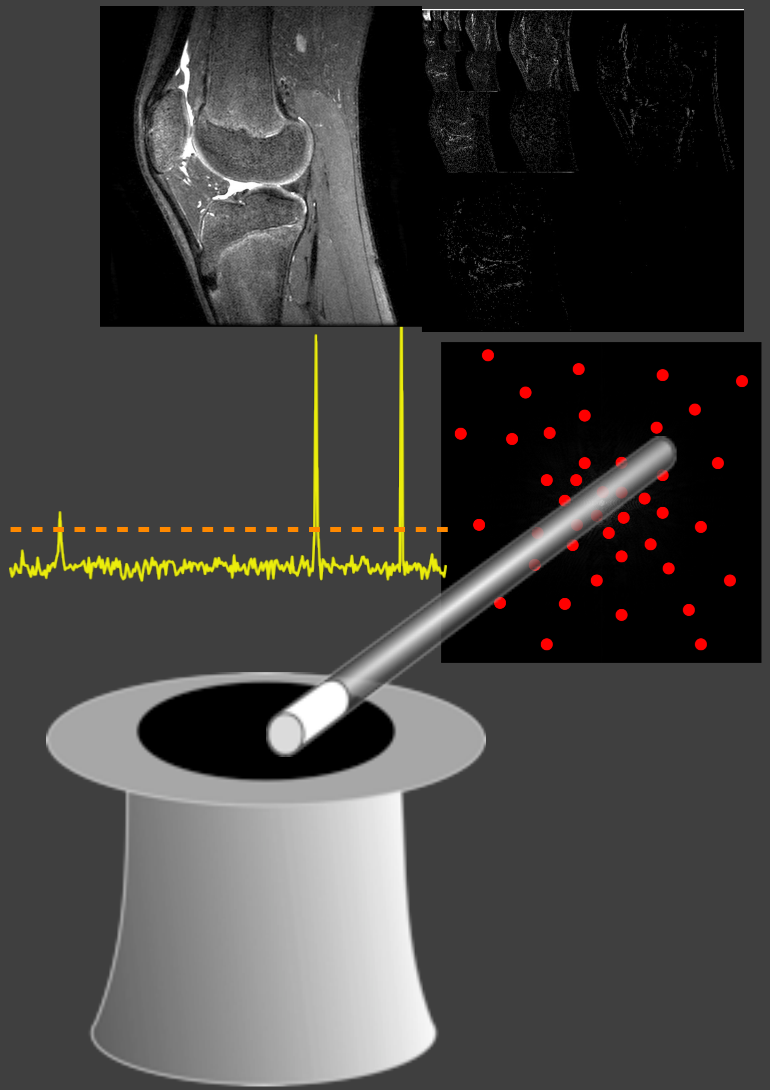
From M. Lustig

# Compressed sensing recipe

1. Sparse signal model

2. Incoherent sensing operator

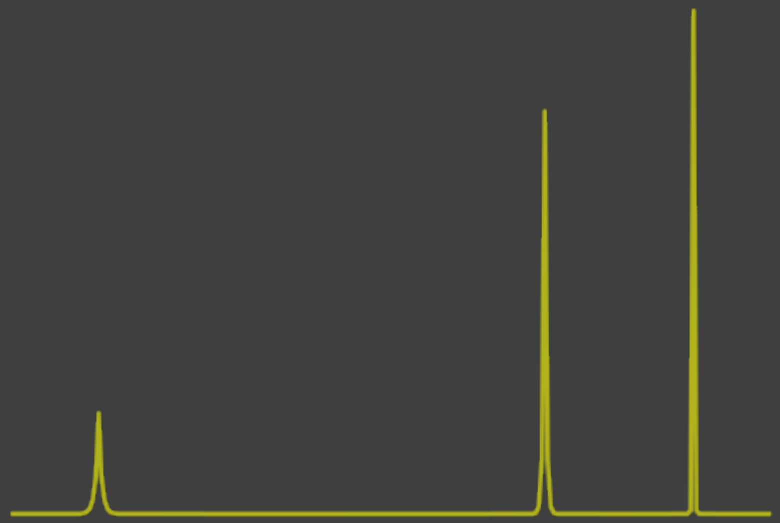
3. Non-linear reconstruction algorithm



# Sparsity vs. Noise

- A signal is sparse if it is mostly zero

Sparse

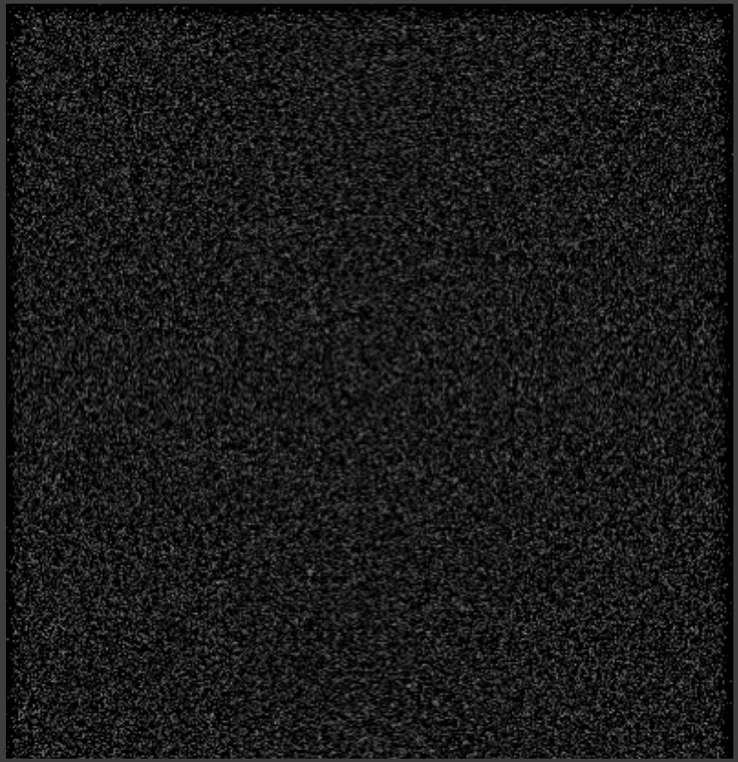


Not sparse

# Sparsity vs. Noise

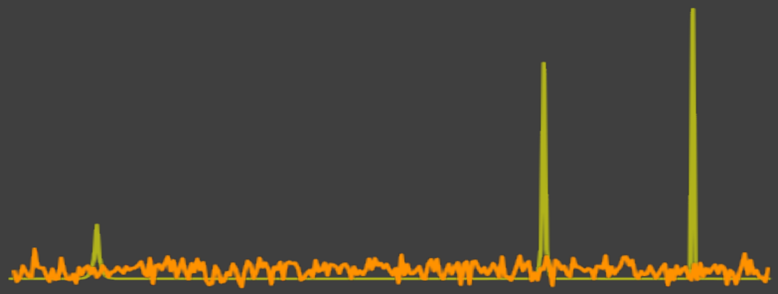
Sparse

Not sparse



From J. Trzasko

# Sparsity vs. Noise

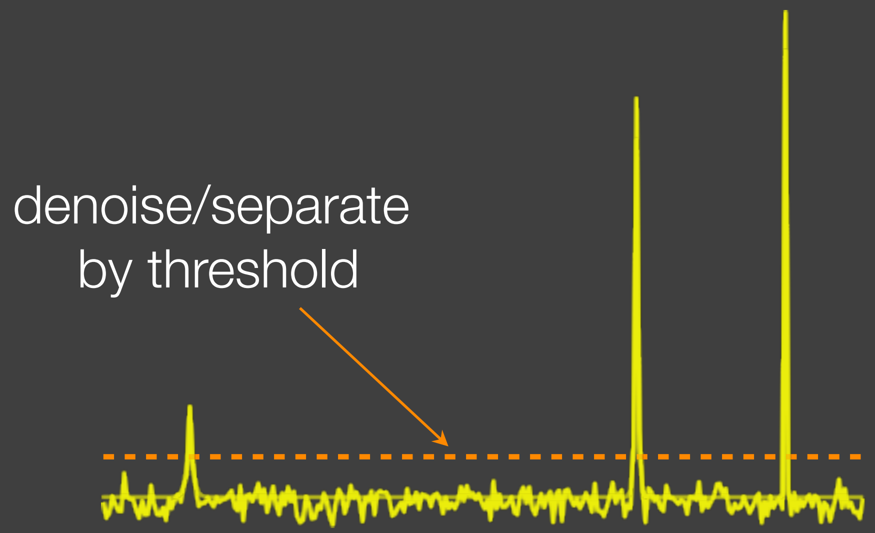


From J. Trzasko

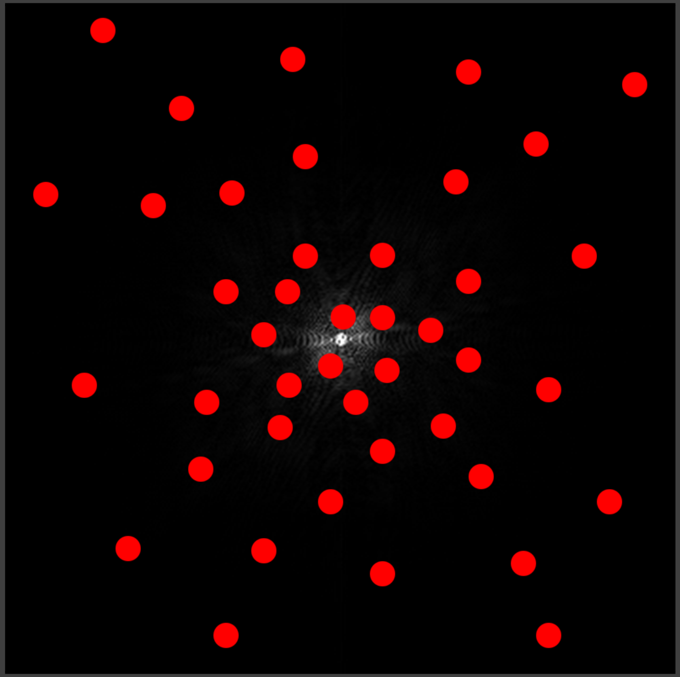
# Sparsity vs. Noise

- To separate sparsity from noise…

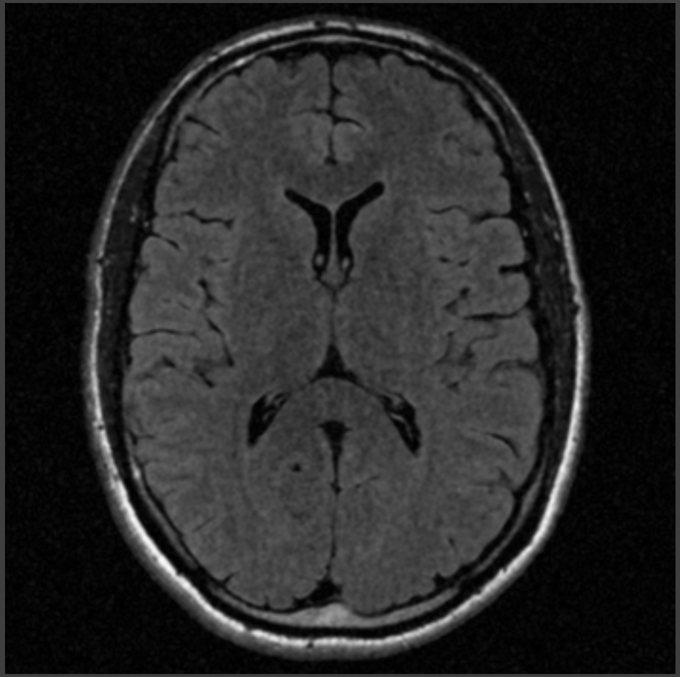
…apply a threshold!




1. Under-sampled k-space



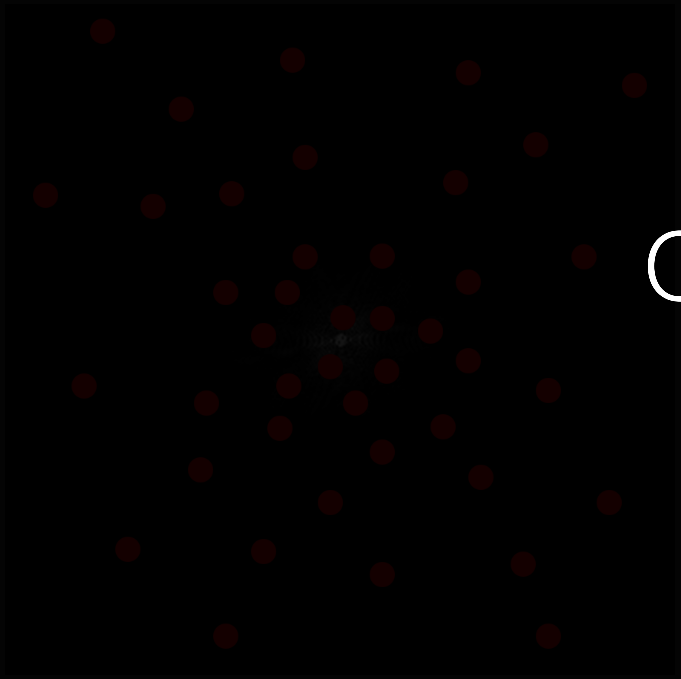
2. Image with noise-like artifacts



(not sparse!)

3. ???

1. Under-sampled k-space



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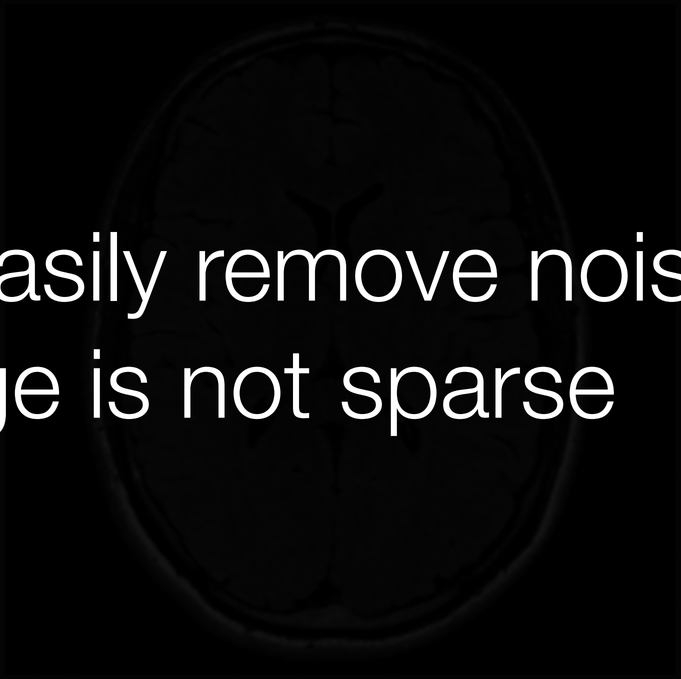


(not sparse!)

3. ???

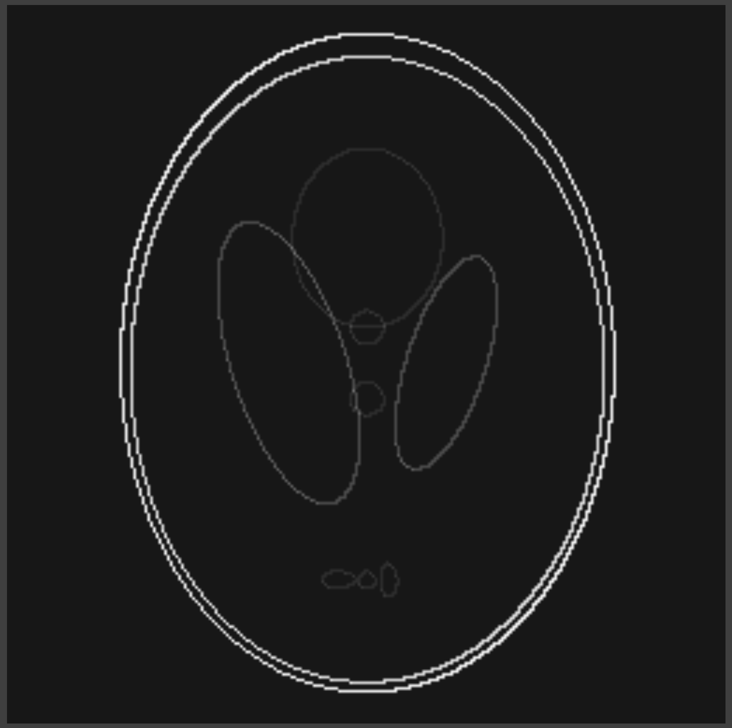
Cannot easily remove noise
if image is not sparse

# Transform Sparsity

- Most medical images are sparse in an alternative representation

not sparse

sparse edges

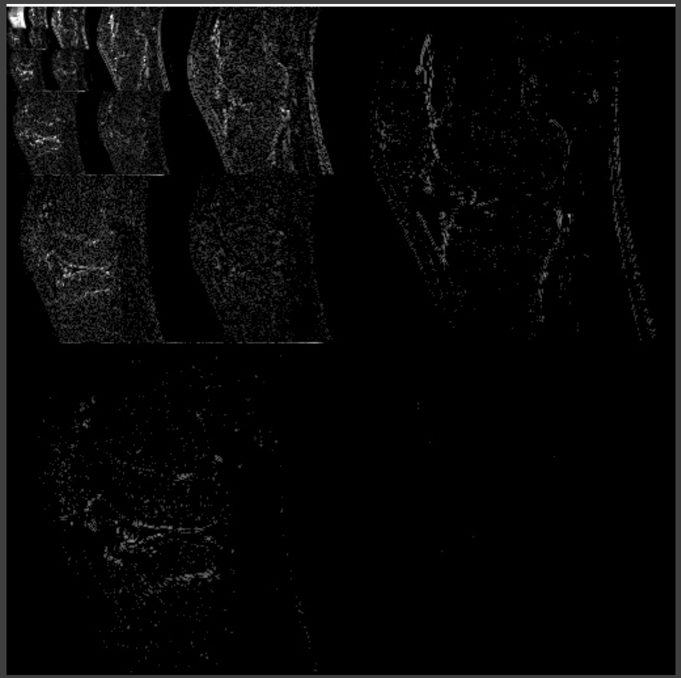


# Transform Sparsity

- Most medical images are sparse in an alternative representation

not sparse

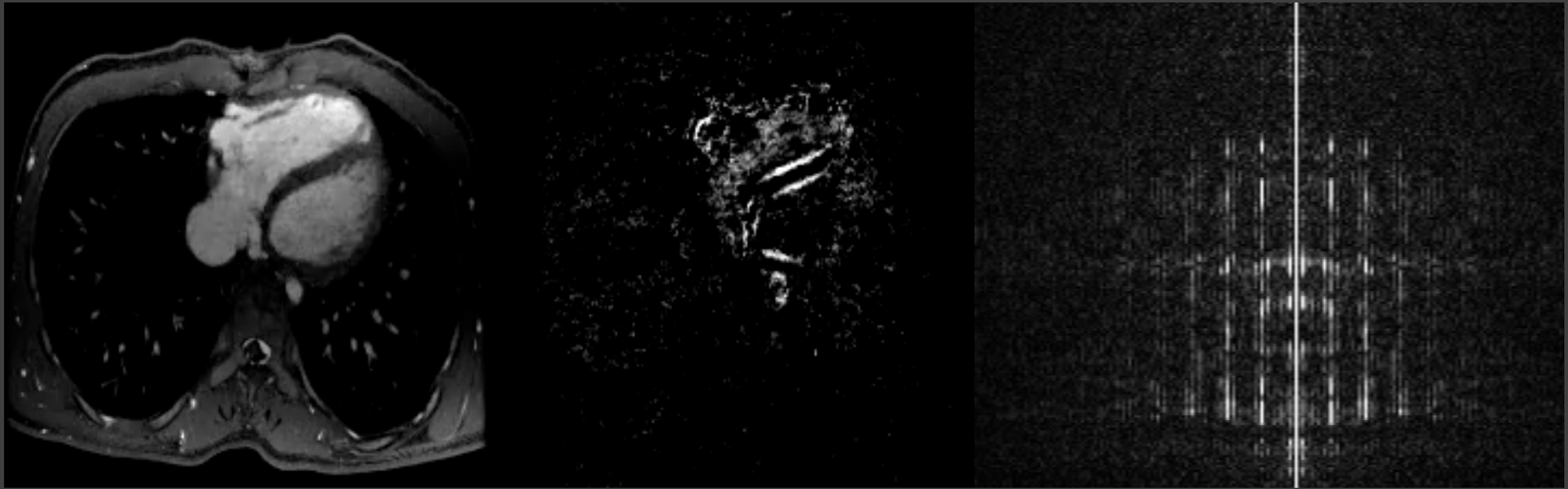
sparse wavelet



# Transform Sparsity

- Most medical images are sparse in an alternative representation

not sparse | sparse temporal finite differences | sparse temporal frequency

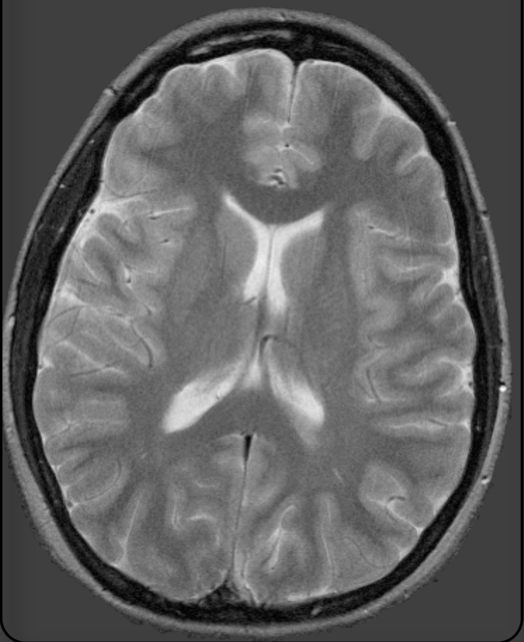


From J. Cheng

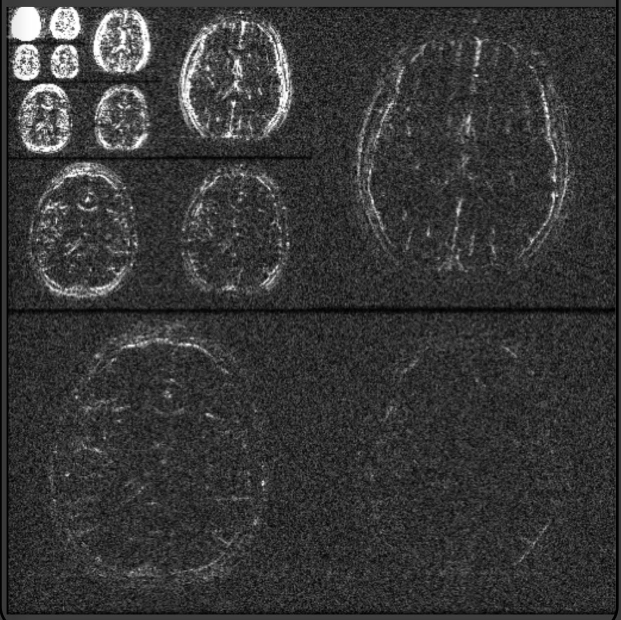
# Transform Sparsity

1. Transform the image to a sparse domain
2. Apply denoising/thresholding
3. Transform back to the image domain

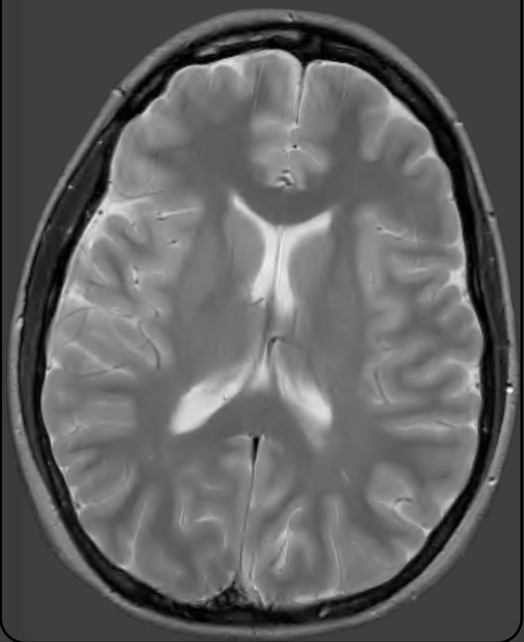
not sparse



sparse

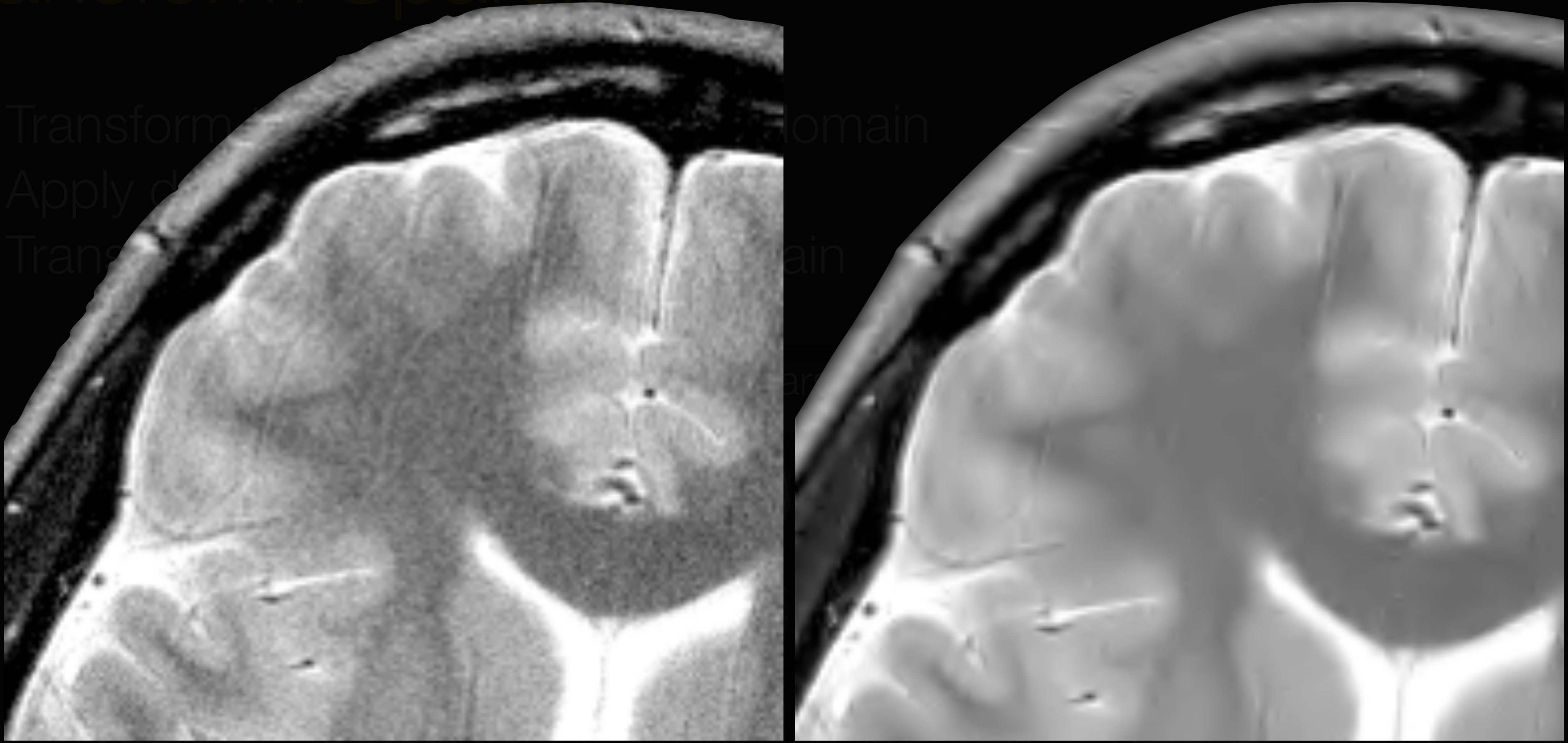


denoised

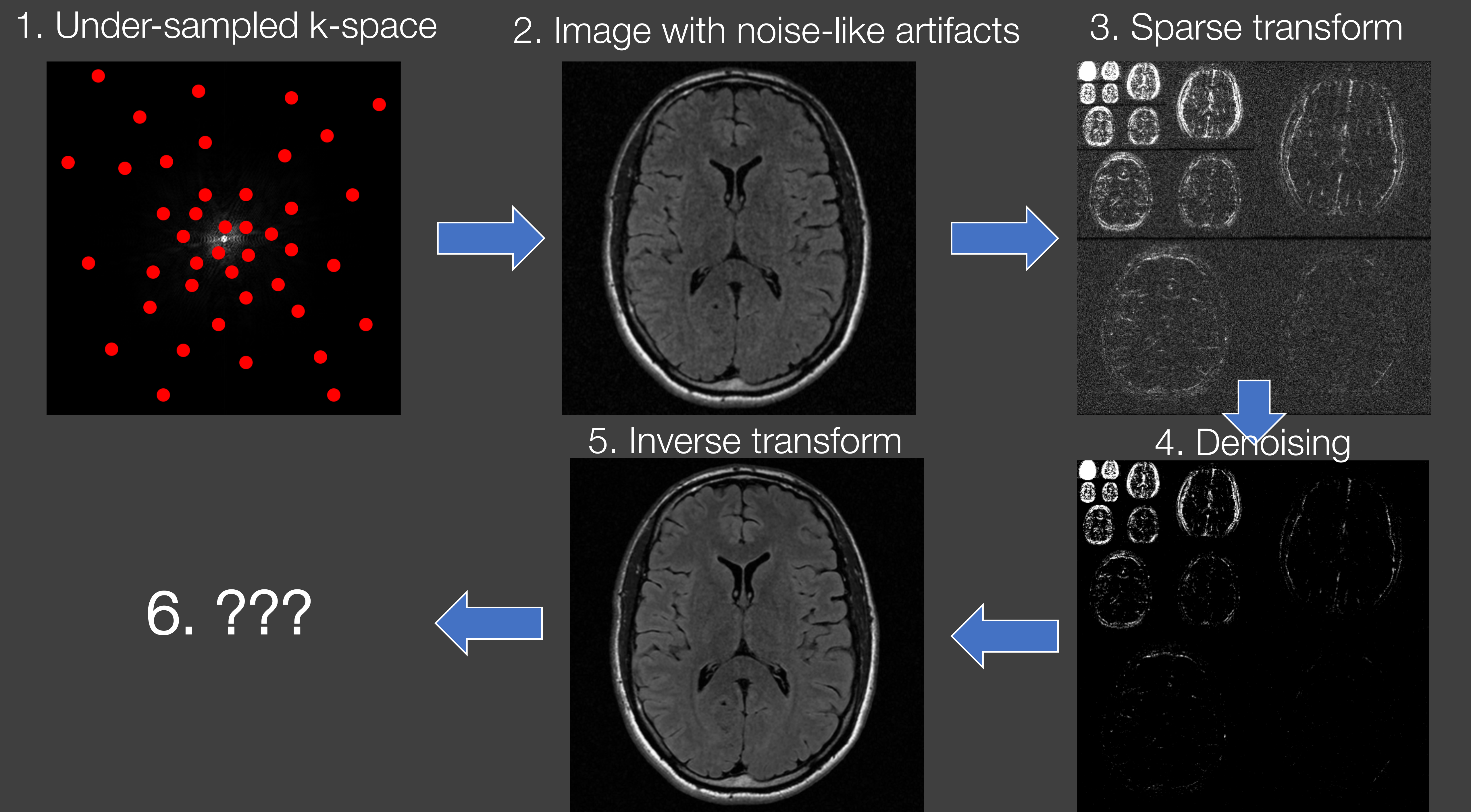


From M. Lustig

# wavelet denoising



From M. Lustig


1. Under-sampled k-space
2. Image with noise-like artifacts
3. Sparse transform
4. Denoising
5. Inverse transform
6. ???


Our new image is not perfectly denoised. Why?

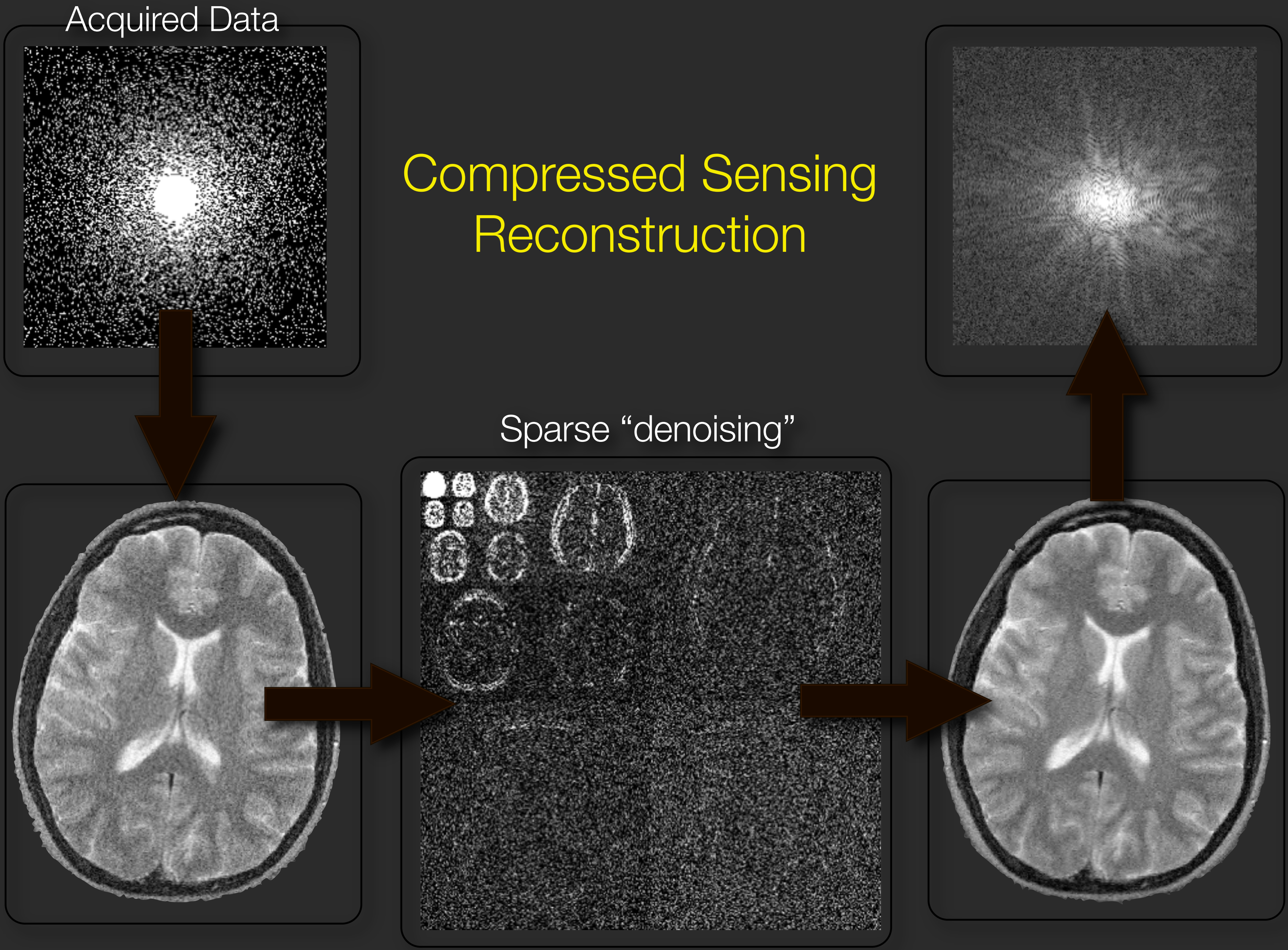
# Iterative noise removal

- “Noise-like” artifacts are not actually due to noise
- They are due to under-sampling of k-space

Intuitive idea: After denoising the image, compare and fuse the new k-space with our acquired k-space.

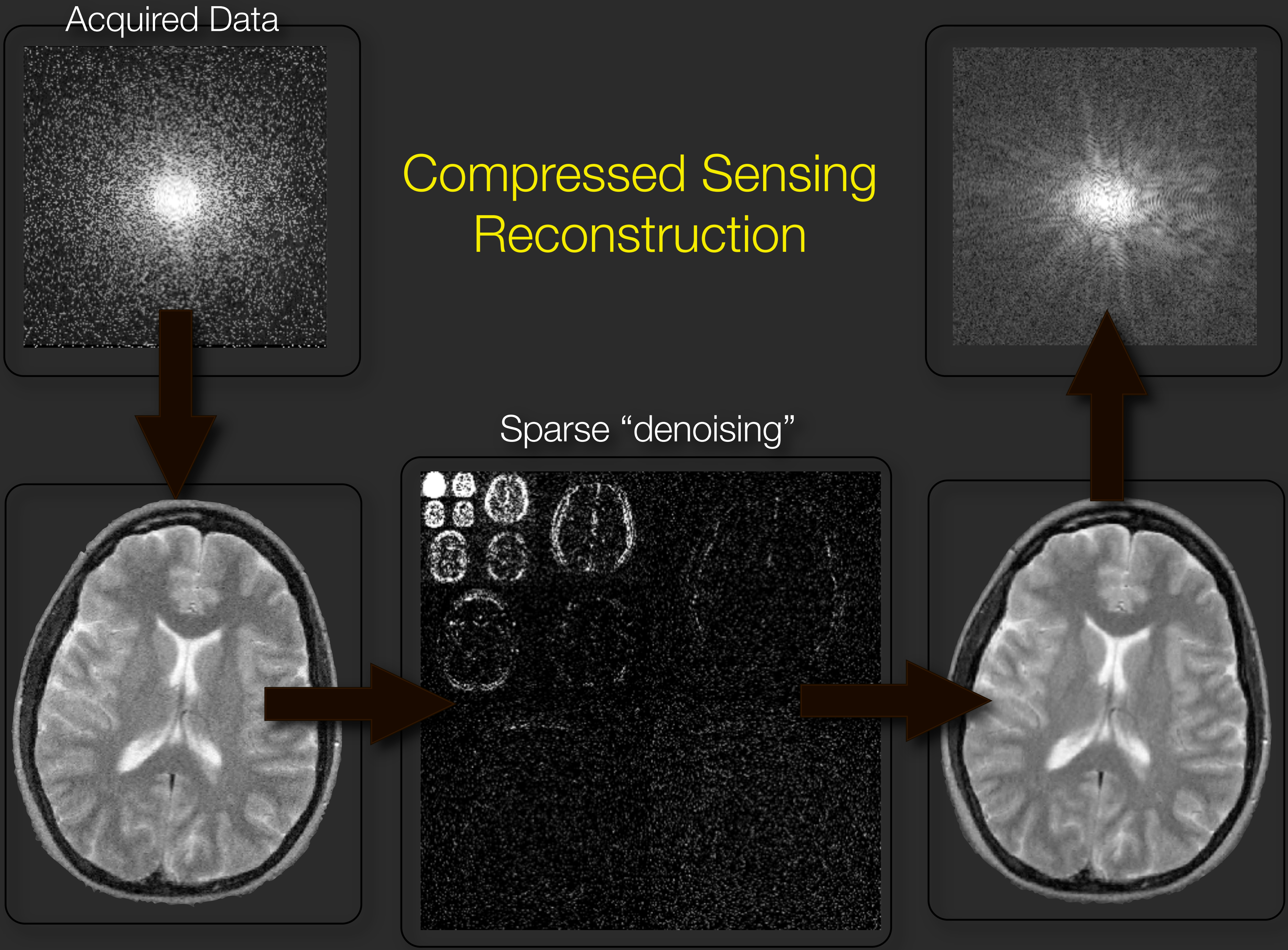
Then re-apply the process

# Compressed Sensing Reconstruction



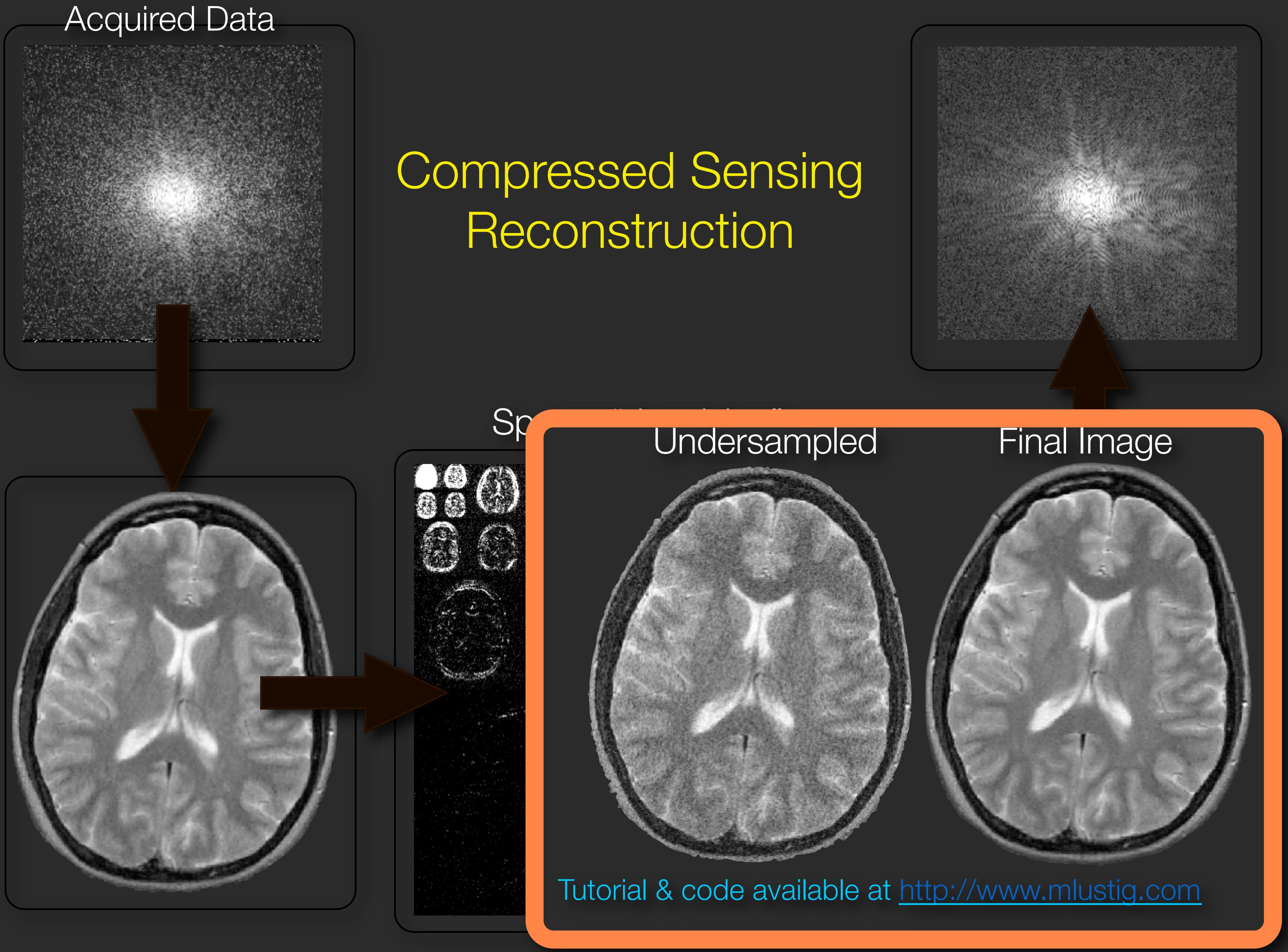

From M. Lustig

# Compressed Sensing Reconstruction




From M. Lustig

# Compressed Sensing Reconstruction




From M. Lustig

# Why random sampling?

- Intuition: random sampling causes noise-like aliasing artifacts
- Theory: want an incoherent sensing operator

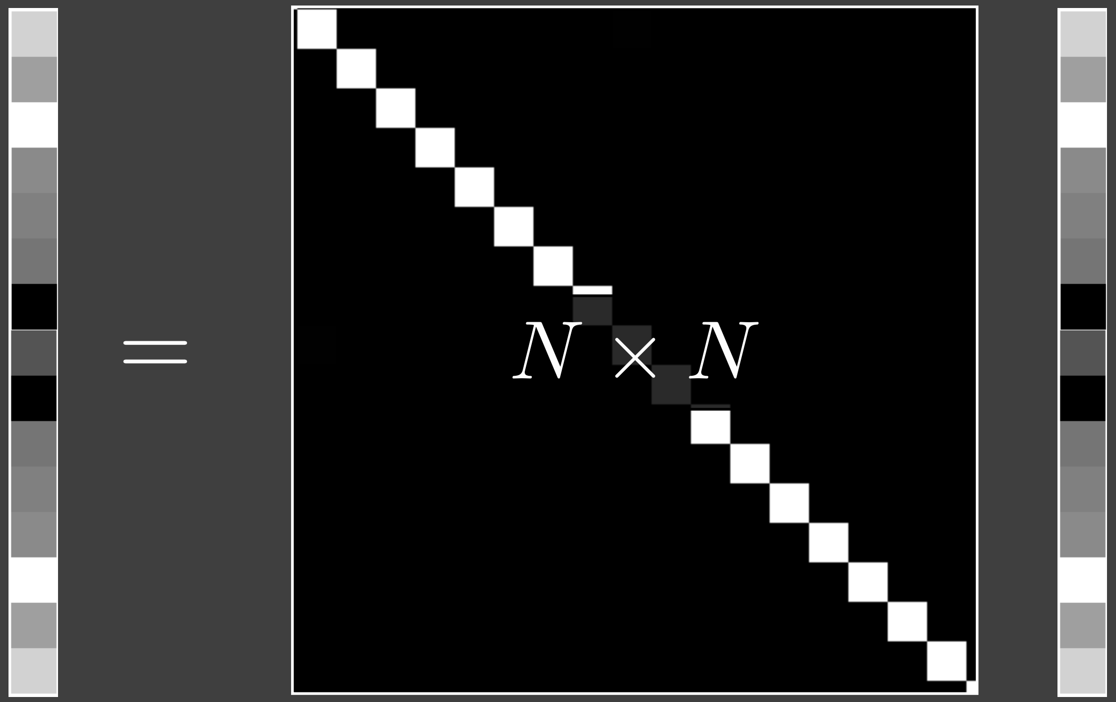
# Traditional sensing

Diagonal system

- Make N linear measurements

$$\mathbf{A} \in \mathbb{C}^{N \times N}$$

$$\mathbf{y} \in \mathbb{C}^N = \mathbf{A}\,\mathbf{x}, \quad \mathbf{x} \in \mathbb{C}^N$$



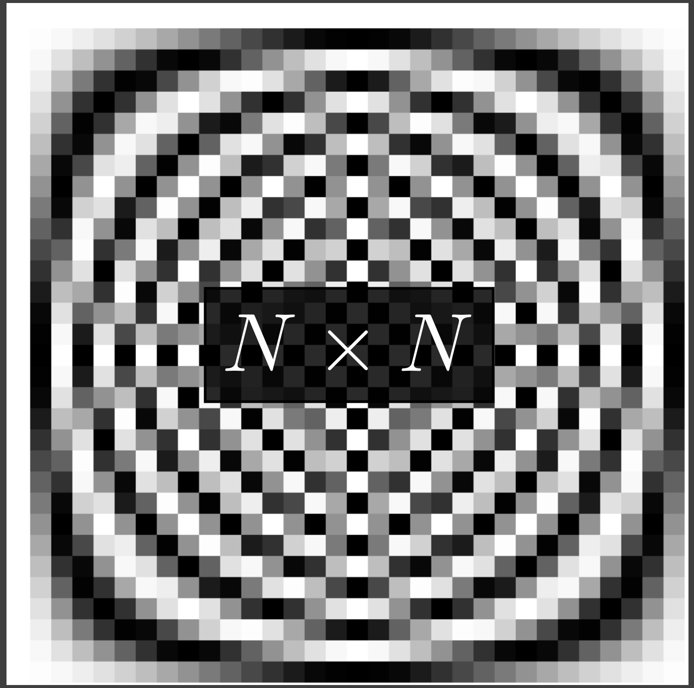

Sensing operator

# Traditional sensing

- Make N linear measurements

Fourier measurements

$$\mathbf{A} \in \mathbb{C}^{N \times N}$$

$\mathbf{y} \in \mathbb{C}^N$  $=$ 

 $\mathbf{x} \in \mathbb{C}^N$

Sensing operator

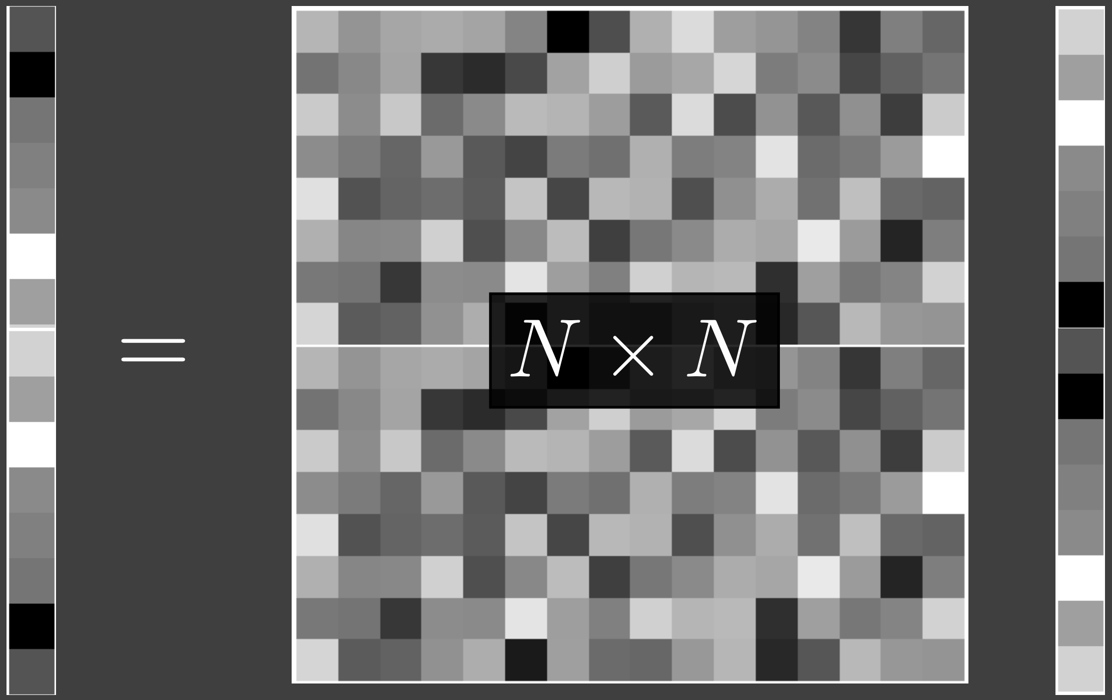
# Traditional sensing

- Make N linear measurements

Random measurements

$$\mathbf{A} \in \mathbb{C}^{N \times N}$$

$$\mathbf{y} \in \mathbb{C}^{N} \quad = \quad \mathbf{x} \in \mathbb{C}^{N}$$




Sensing operator

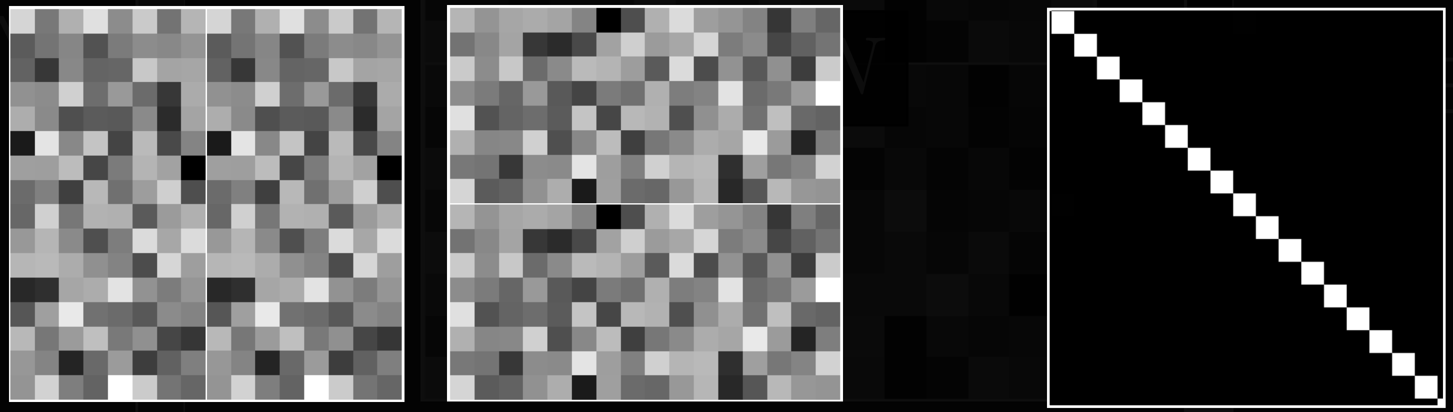
# Traditional sensing

- Make N linear measurements

Fourier measurements

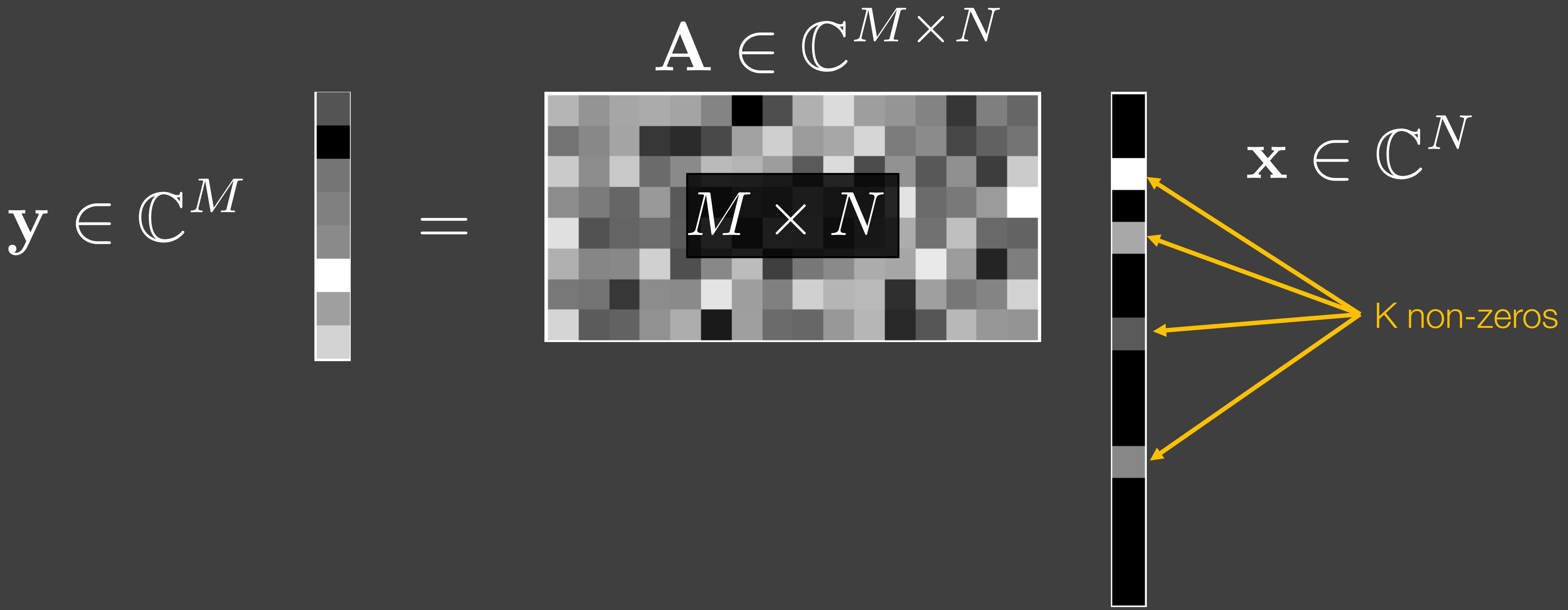
A “good” sensing matrix is orthogonal

$$\mathbf{A}^H \mathbf{A} = \mathbf{I}$$



# Compressed sensing

- Assumption: x is a K-sparse signal (K << N)
  - Make M (K < M < N) incoherent linear measurements

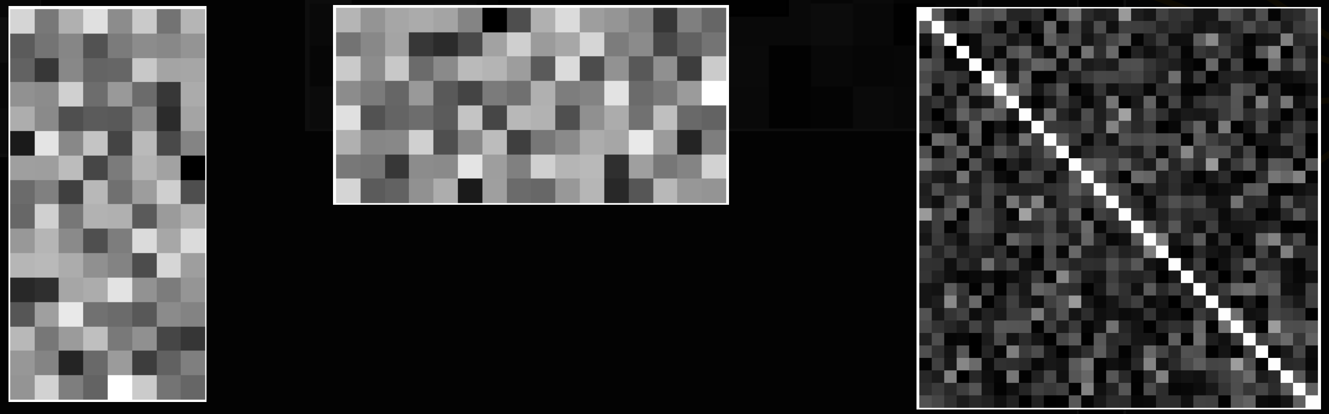


# Compressed sensing

- Assumption: x is a K-sparse signal (K << N)
  - Ma

A "good" compressed sensing matrix is incoherent,
i.e. approximately orthogonal

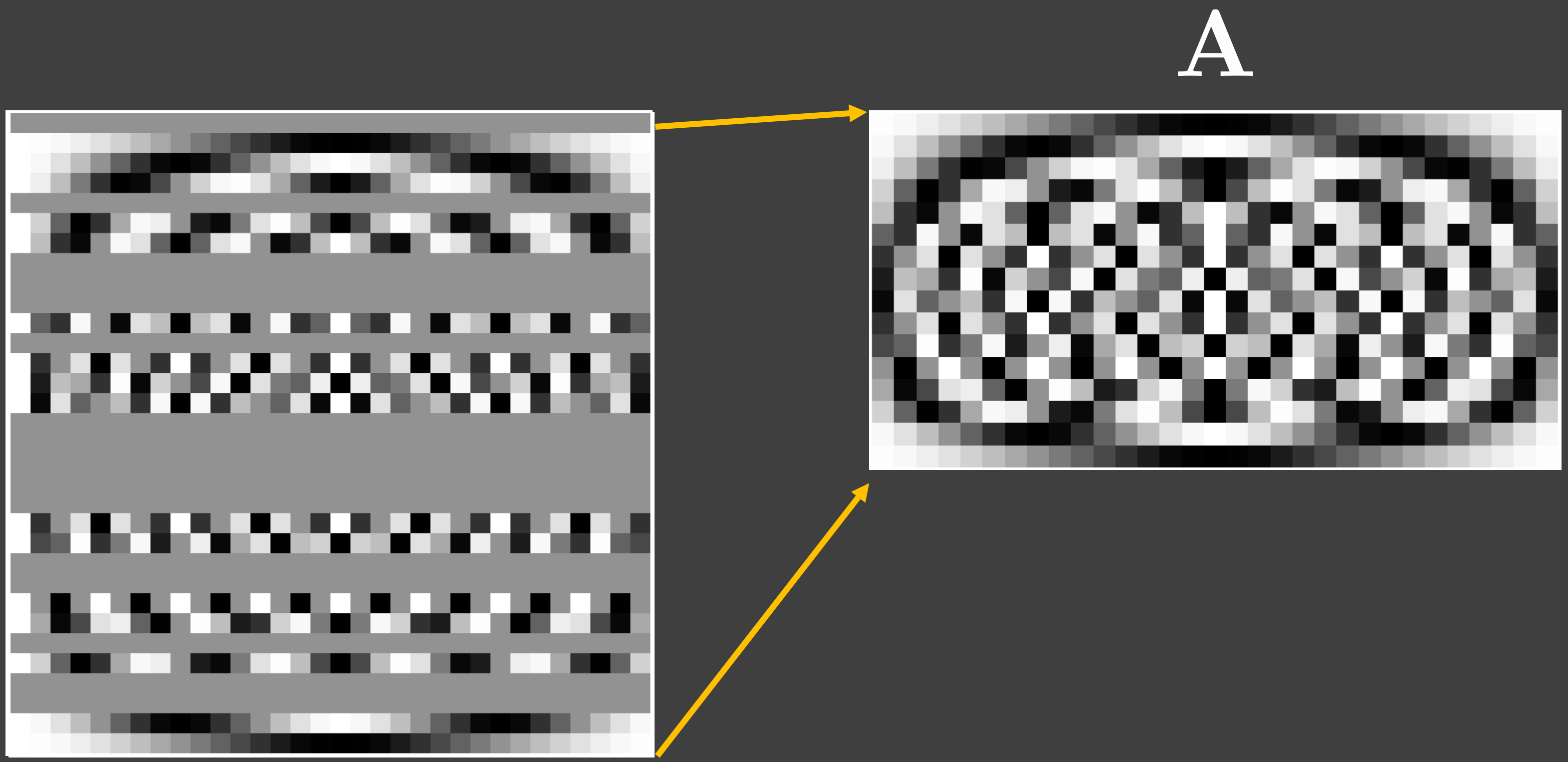
$$\mathbf{A}^H \mathbf{A} \approx \mathbf{I}$$



Incoherency preserves information

# Fourier compressed sensing

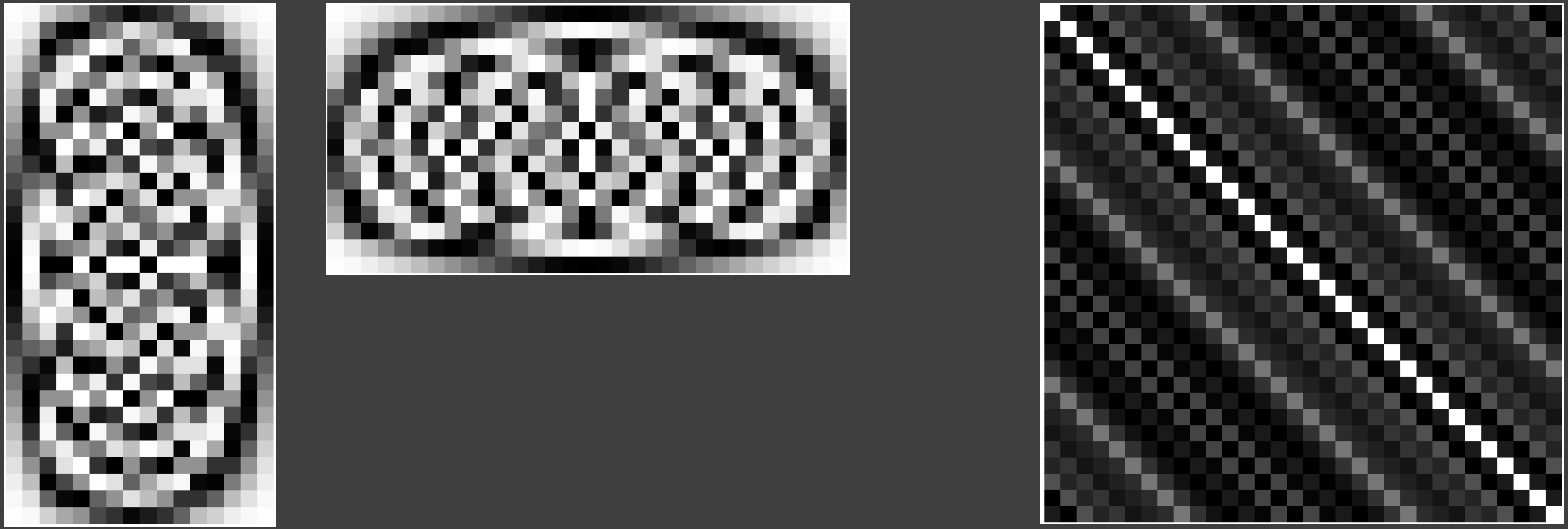
- Randomly under-sampled Fourier matrix:



# Fourier compressed sensing

- Randomly under-sampled Fourier matrix: incoherent

$$\mathbf{A}^H \quad \mathbf{A} \quad \approx \quad \mathbf{I}$$

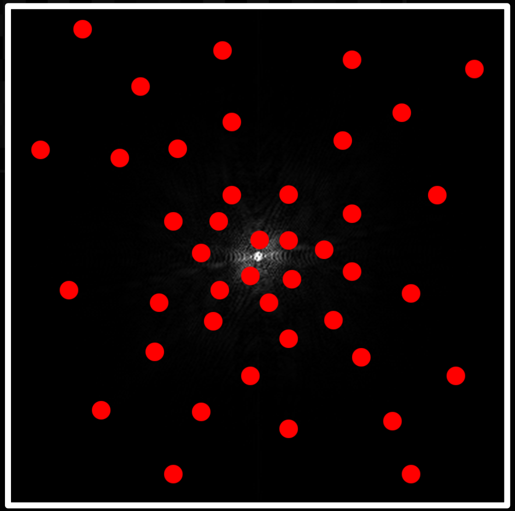


# Fourier compressed sensing

- Randomly under-sampled Fourier matrix: incoherent

Randomly under-sampled k-space

Variable-density: better matches image statistics



# Compressed sensing reconstruction

- Intuition: alternate between denoising (thresholding) and data consistency
- Theory: Solve non-linear, iterative inverse problem
  - L1-minimization promotes sparse solutions

$$||\mathbf{x}||_1 = \sum_i |x_i|$$

**Basis pursuit denoising**

$$\min_{\mathbf{x}} \quad ||\mathbf{T}\mathbf{x}||_1$$
$$\text{subject to} \quad ||\mathbf{y} - \mathbf{A}\mathbf{x}||_2 \leq \epsilon$$

S Chen, D Donoho. “Basis Pursuit,” Technical Report, Department of Statistics, Stanford University.

# Compressed sensing reconstruction

- Intuition: alternate between denoising (thresholding) and data consistency
- Theory: Solve non-linear, iterative inverse problem
  - L1-minimization promotes sparse solutions

$$||\mathbf{x}||_1 = \sum_i |x_i|$$

**Lasso**

$$\min_{\mathbf{x}} \frac{1}{2}||\mathbf{y} - \mathbf{Ax}||_2 + \lambda||\mathbf{Tx}||_1$$

Tibshirani, R. (1996). Regression shrinkage and selection via the lasso. J. Royal. Statist. Soc B., Vol. 58, No. 1, pages 267-288).

# Compressed sensing reconstruction

- Intuition: alternate between denoising (thresholding) and data consistency
- Theory: Solve non-linear, iterative inverse problem
  - L1-minimization promotes sparse solutions
- Application: Combine with parallel imaging and non-Cartesian sampling

# BART – MRI reconstruction toolbox

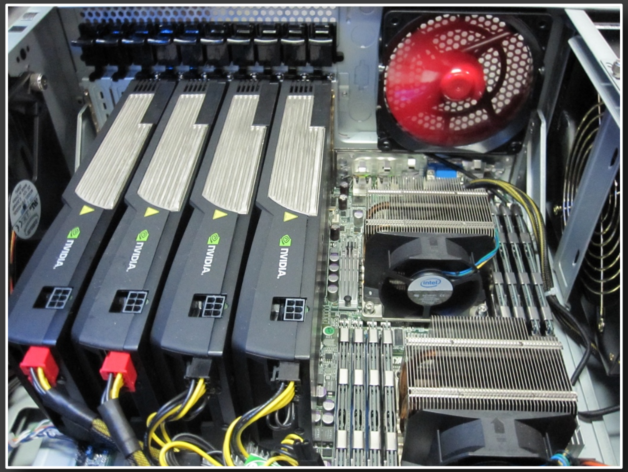
- Software framework for CS MRI
  - Implements parallel imaging and CS
  - Built in parallelism (CPU/GPU)

- Emphasis on
  - Rapid prototyping
  - Clinical feasibility / robustness
  - Collaboration

- Open source, BSD license
  http://mrirecon.github.io/bart/

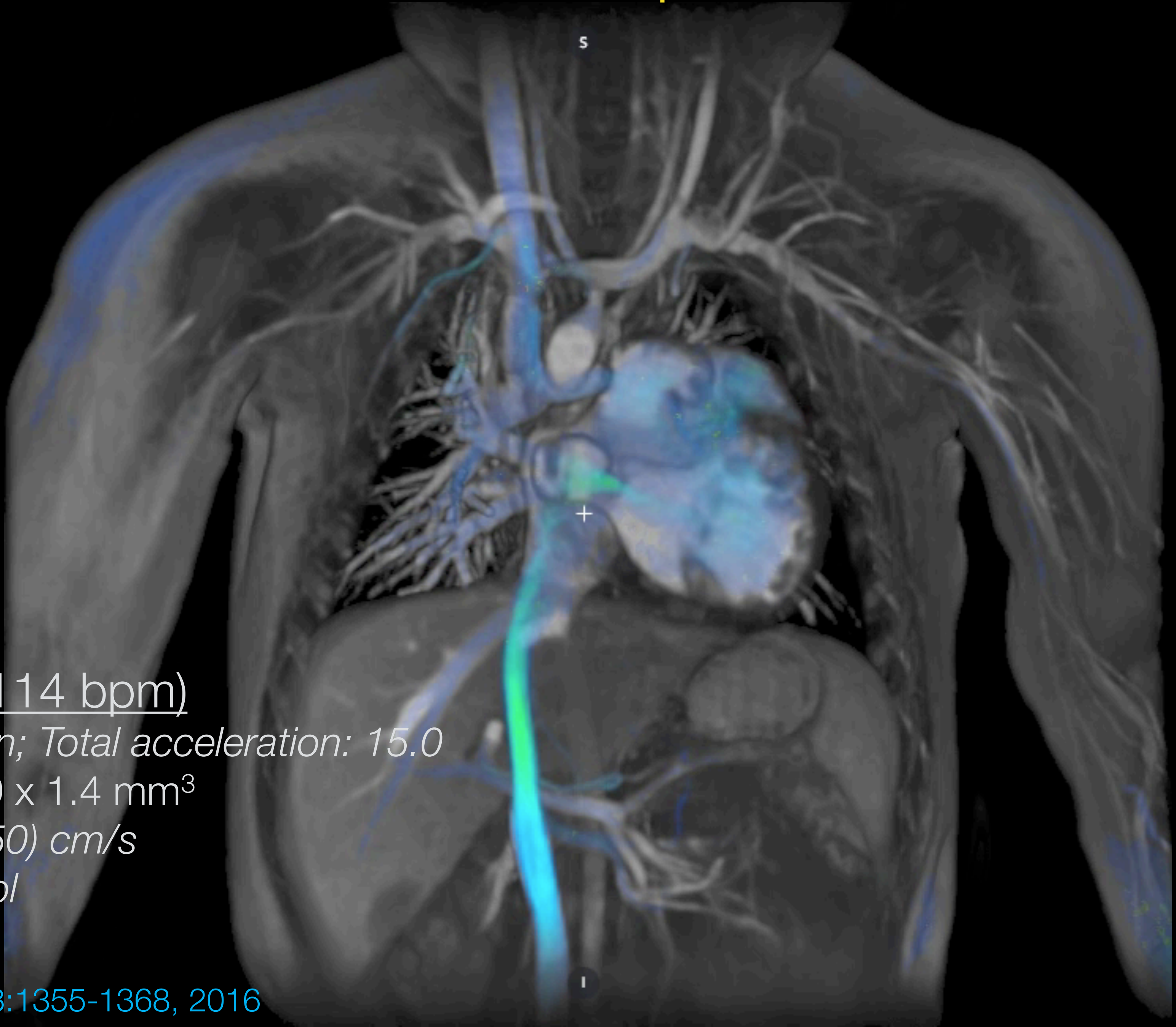
UCB alumni, now at Göttingen University

Prof. Martin Uecker



*Development supported by NIH, GE healthcare and by personal donation from David Donoho

# Cardiac-resolved volumetric phase contrast MRI (4D Flow)




12 month male (114 bpm)

*Scan time: 11:05 min; Total acceleration: 15.0*

Resolution: 0.9 x 0.9 x 1.4 $mm^3$

*VENC: (150, 150, 150) cm/s*

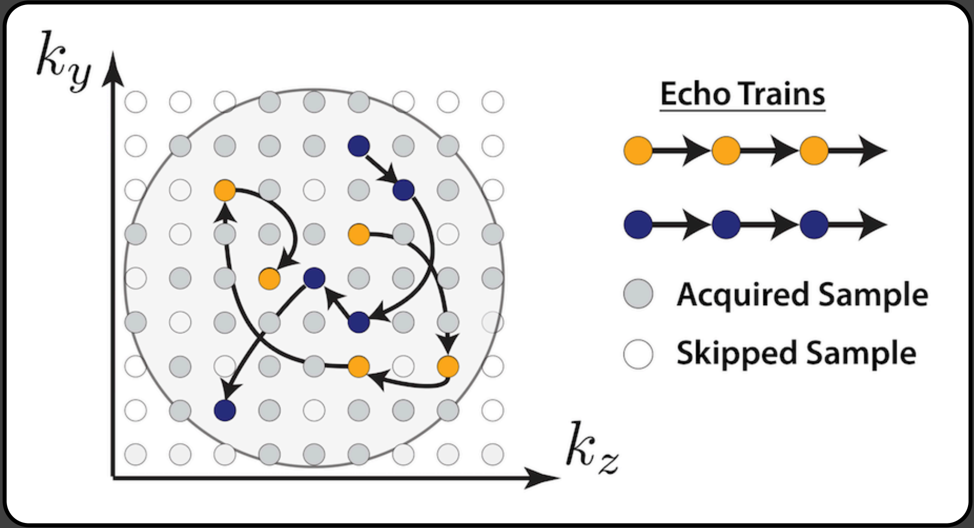
*Contrast: ferumoxytol*

JY Cheng, et al., JMRI 43:1355-1368, 2016

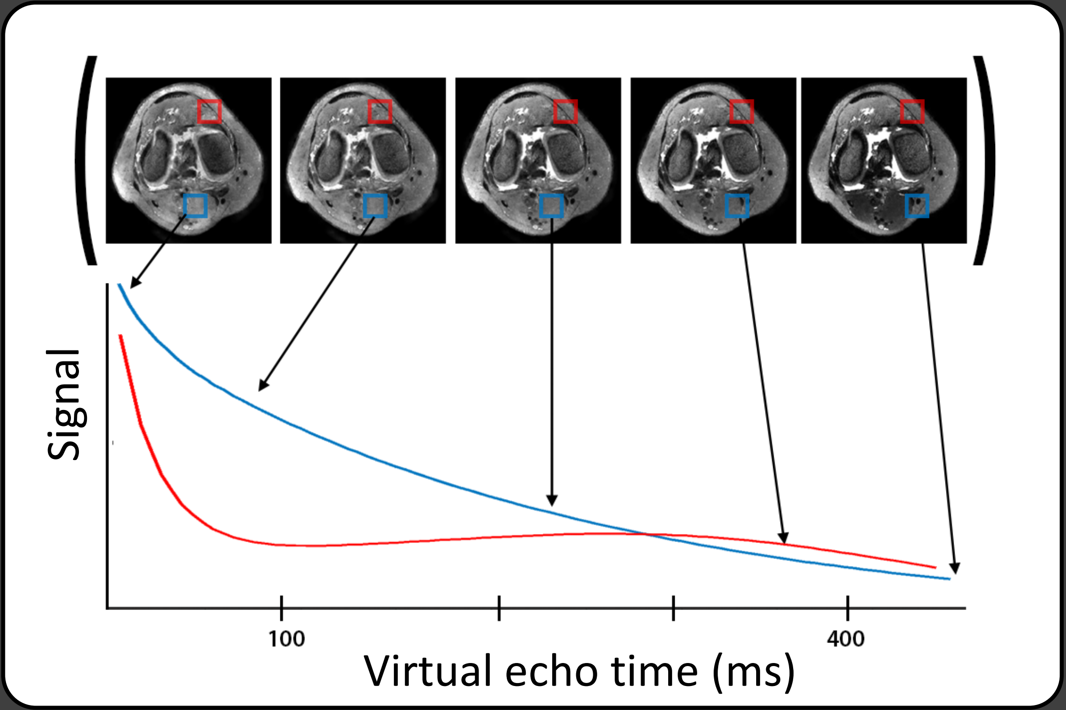
powered by BART

# Multi-contrast 3D FSE

Randomly shuffled echo trains




Compressed sensing in relaxation dimension



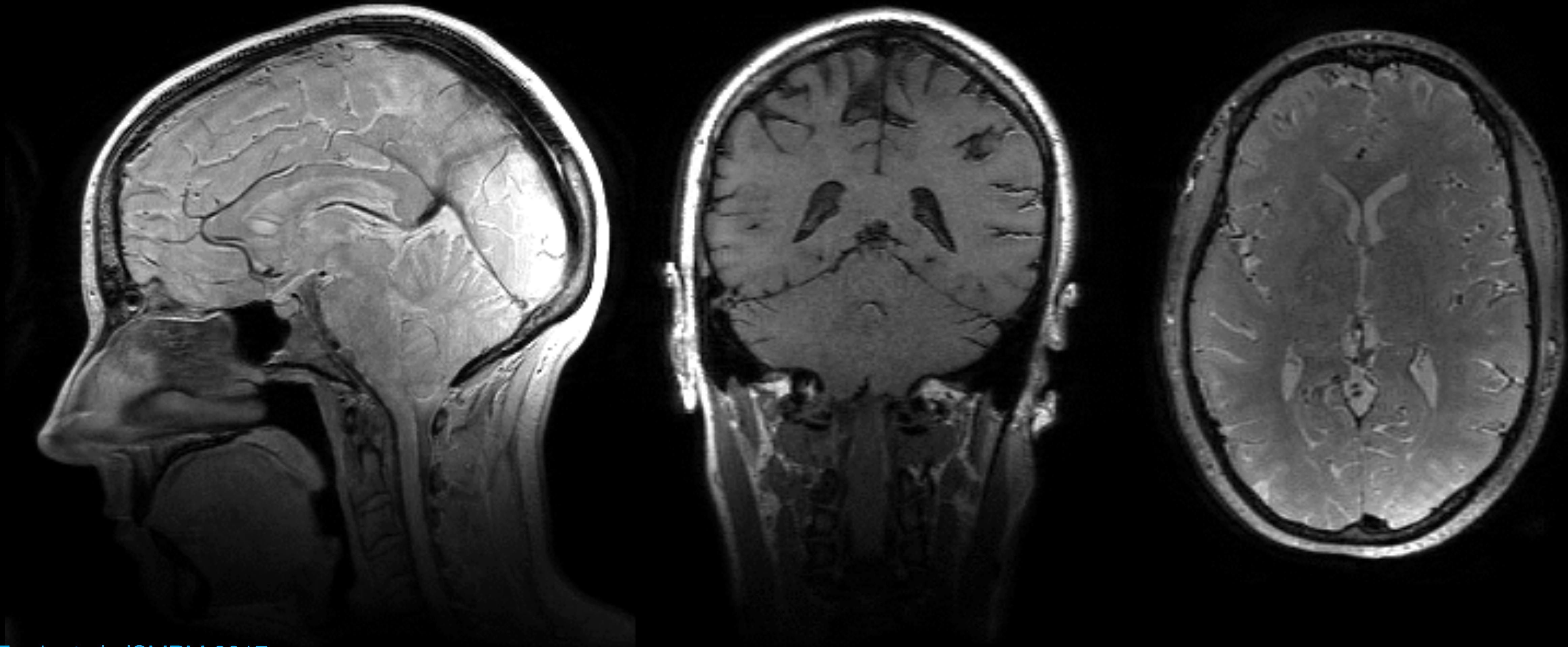

Volumetric, multi-contrast reconstruction

powered by BART

J.I. Tamir et al., MRM 2016. doi: 10.1002/mrm.26102

# Multi-contrast 3D FSE

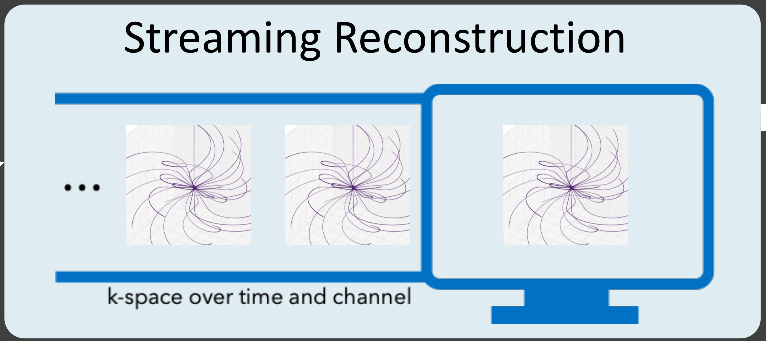
Scan time: 7 minutes
Resolution: 0.8 x 0.8 x 1.2 $mm^3$



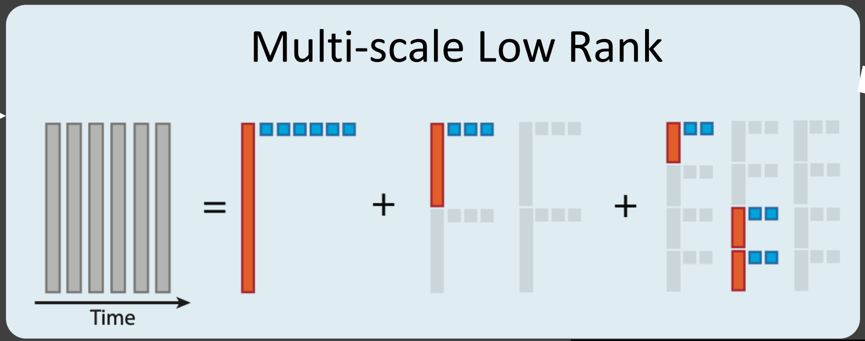
J.I. Tamir et al., ISMRM 2017.

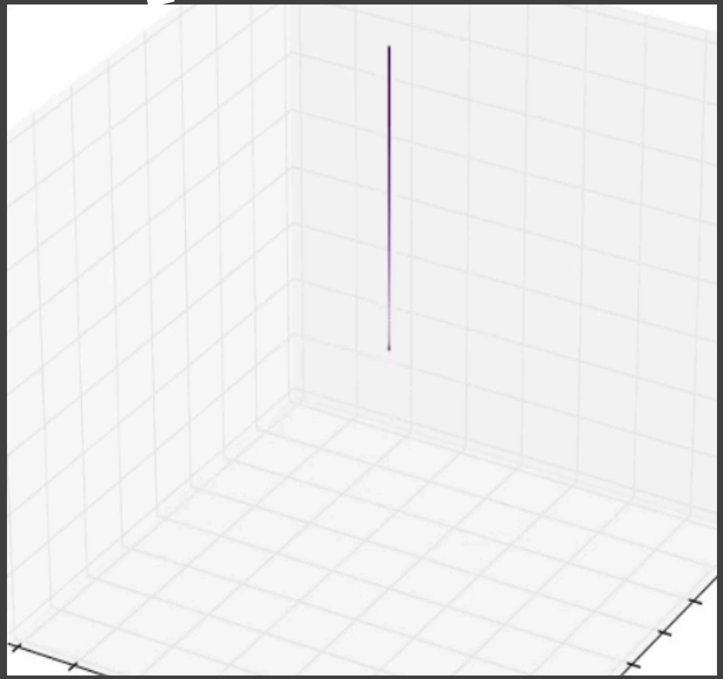
# Extreme MRI: real-time dynamic imaging

1 NUFFT per iteration




1GB Compressed Representation



2GB k-space

Abstract # 1176
Multidimensional Signal Encoding Decoding
Thursday, 16 May 2019
Room 710B

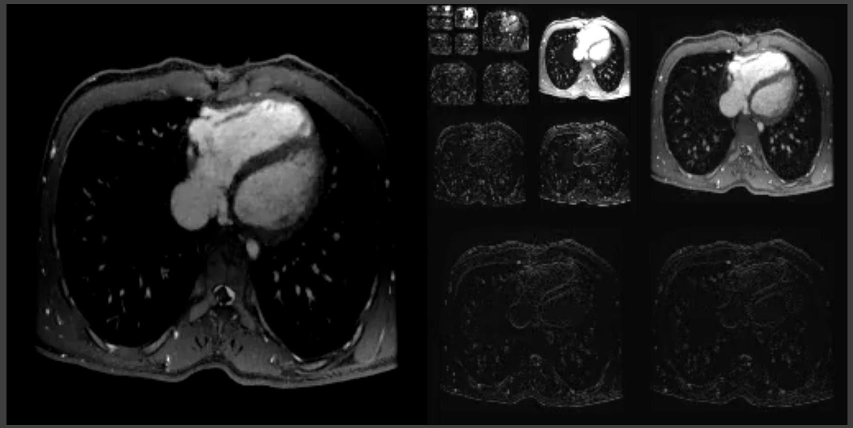
| | |
|---|---|
| **Scan time:** | 4m 40s |
| **Temporal Res** | 580 ms |
| **Matrix Size** | 392 x 318 x 165 |
| **Spatial Res** | 1 x 1 x 2.8mm$^3$ |

100GB Image

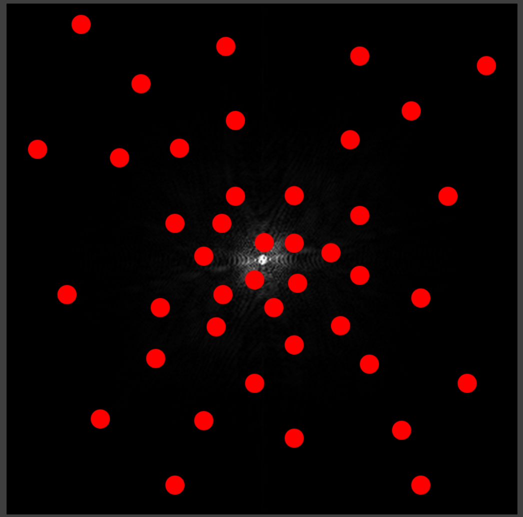
From Frank Ong

# Compressed Sensing MRI

1. Find a sparse transform representation
   - Apply spatially, temporally, …



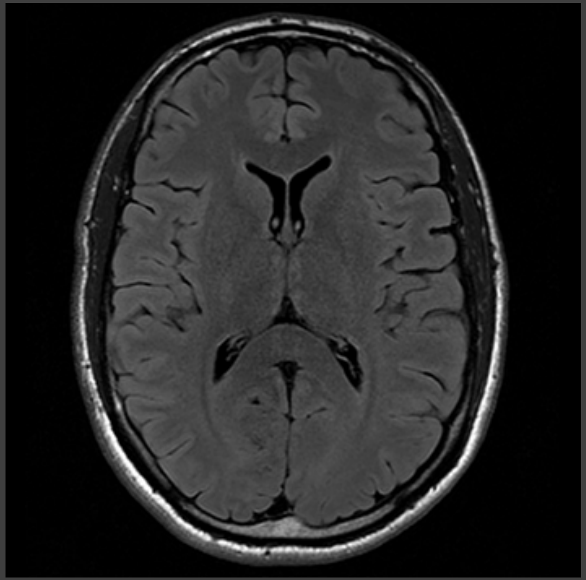
2. Sample k-space incoherently (random)
   - Make artifacts look like noise



3. Reconstruct using sparsity-promoting iterative algorithm

# Challenges

- Requires modification of the sequence
- Increased noise because of less data
  - Unrelated to “noise-like” artifacts
- Artifacts from sparse denoising
  - Blocking artifacts, over-smoothing, temporal blurring
- Difficulty choosing denoising threshold
- Increased computational complexity in reconstruction



Blocking artifacts

Over-smoothing

Slides/images
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www.jtsense.com